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MATHEMATICAL MODELLING OF A THERMALLY STIMULATED OIL RESERVOIR -- HOT WATER INJECTION

BY

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE
IN
PETROLEUM ENGINEERING

FACULTY OF ENGINEERING

DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING

EDMONTON, ALBERTA

JUNE, 1967

THE UNIVERSITY OF ALBERTA

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© A. RICHARD THACHUK

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled MATHEMATICAL MODELLING OF A THERMALLY STIMULATED OIL RESERVOIR -- HOT WATER INJECTION, submitted by A. R. Thachuk in partial fulfilment of the requirements for the degree of Master of Science in Petroleum Engineering.

ABSTRACT

Three thermal recovery processes are discussed: (1) cyclic or intermittent steam injection, (2) steam displacement, and (3) hot-water injection.

An approximate mathematical model describing a steam displacement process for a radial system was formulated but not solved. The system of five non-linear partial differential equations with variable coefficients describes the simultaneous flow of three fluids, oil, water, and steam under the influence of a pressure and thermal gradient. The heat losses to the overburden and underburden were included in the formulation.

A second mathematical model was developed to describe a linear hot-water injection process. The simultaneous flow of two fluids, oil and water, under the influence of a pressure and a thermal gradient were considered. The liquids were treated as compressible and the heat losses to the overburden and underburden were neglected.

The hot-water injection model was solved using numerical methods. The alternating direction explicit procedure (ADEP) and the Newton-Raphson iterative procedure were used to solve the hydrodynamical equations. Crank-Nicolson finite difference equations and the square root method for inverting a matrix, extended to non-symmetric matrices, were employed in the solution of the thermal energy equation.

Pressure, saturation and temperature distributions along with cumulative oil production histories and a material balance check on the results are presented. A discussion of convergence problems encountered with the computer program is included.

The alternating direction explicit procedure (ADEP) yielded a stable, convergent, computationally and economically feasible solution to a one-dimensional, compressible, isothermal, two phase flow problem.

Secondly, mathematical modelling of a thermally stimulated reservoir model is possible by the method outlined in this work and the results show that

increased recoveries and lower injection pressures are characteristic of highly temperature sensitive oil viscosity models when subjected to thermal stimulation.

Although a verification of the model was not possible the general shape of the pressure distribution, saturation distribution, temperature distribution and cumulative oil production history curves are representative of those presented in the literature.

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TABLE OF CONTENTS

Page

LIST OF FIGURES (i)

INTRODUCTION 1

THEORY AND LITERATURE REVIEW -- STEAM INJECTION -- THREE PHASE FLOW 5

THEORY AND LITERATURE REVIEW -- HOT-WATER INJECTION -- TWO PHASE FLOW 17

DISCUSSION OF POSSIBLE METHODS OF SOLUTION OF THE HOT-WATER INJECTION MODEL 25

DISCUSSION OF CONVERGENCE AND STABILITY OF THE COMPUTER PROGRAM 33

DISCUSSION OF RESULTS 36

CONCLUSIONS 70

RECOMMENDATIONS 72

NOMENCLATURE 74

LIST OF REFERENCES 78

APPENDIX A

Derivation of the Steam Displacement Mathematical Model 84

APPENDIX B

Derivation of the Hot-Water Injection Model 95

APPENDIX C

Discretization of Partial Differential Equations and Methods of Solution 102

Alternating Direction Explicit Procedure (ADEP) -- One-Dimension 102

Page

Newton-Raphson Iterative Procedure 113

The Thermal Energy Equation -- Crank-Nicholson Approximation 119

APPENDIX D

Comparison of the "Open-Flow" and "No-Flow" Boundary Problems 126

APPENDIX E

Discussion of Validity of Darcy's Law for Nonisothermal Systems 133

Validating the use of an Effective Thermal Conductivity in the Thermal Energy Equation 134

APPENDIX F

Reservoir and Fluid Properties -- Computer Input Parameters 136

APPENDIX G

The Computer Program 143

Computer Program Nomenclature 146

Sample Computer Input - Data Cards 152

The Computer Program 153

Computer Output I 165

Computer Output II 171

Computer Output III 177

LIST OF FIGURES

Figure No.		Page
1	Diagram of the Physical System -- Steam Displacement	8
2	Diagram of the Physical System -- Hot Water Injection	18
3	Diagram of "No-Flow" and "Open-Flow" Boundaries for Reservoir Models	19
4-5-6	Pressure Distributions	50-52
7	Saturation Distributions	53
8	Pressure Distributions $\surd=0.56$, $\Delta x=0.300$ ft., $\Delta t=0.015$ Hrs.	54
9	Saturation Distributions $\surd=0.56$, $\Delta x=0.300$ ft., $\Delta t = 0.015$ Hrs.	55
10	Temperature Distributions $\surd=0.56$, $\Delta x=0.300$ ft., $\Delta t = 0.015$ Hrs.	56
11	Cumulative Oil Production History $\surd=0.56$, $\Delta x=0.300$ ft., $\Delta t=0.015$ Hrs.	57
12	Cumulative Material Balances $\surd=0.56$, $\Delta x=0.300$ ft., $\Delta t=0.015$ Hrs.	58
13	Pressure Distributions $\surd=1.12$, $\Delta x=0.300$ ft., $\Delta t=0.020$ Hrs.	59
14	Saturation Distributions $\surd=1.12$, $\Delta x=0.300$ ft., $\Delta t=0.020$ Hrs.	60
15	Temperature Distributions $\surd=1.12$, $\Delta x=0.300$ ft., $\Delta t=0.020$ Hrs.	61
16	Cumulative Oil Production History $\surd=1.12$, $\Delta x=0.300$ ft., $\Delta t=0.020$ Hrs.	62
17	Cumulative Material Balances $\surd=1.12$, $\Delta x=0.300$ ft., $\Delta t=0.020$ Hrs.	63

Figure No.		Page
18	Pressure Distributions $\surd=1.50$, $\Delta x=0.300$ ft., $\Delta t=0.015$ Hrs.	64
19	Saturation Distributions $\surd=1.50$, $\Delta x=0.300$ ft., $\Delta t=0.015$ Hrs.	65
20	Temperature Distributions $\surd=1.50$, $\Delta x=0.300$ ft., $\Delta t=0.015$ Hrs.	66
21	Cumulative Oil Production History $\surd=1.50$, $\Delta x=0.300$ ft., $\Delta t=0.015$ Hrs.	67
22	Cumulative Material Balances $\surd=1.50$, $\Delta x=0.300$ ft., $\Delta t=0.015$ Hrs.	68
23	Pressure Distributions (Sketch Only) (Reference 35)	69
C-1	One-Dimensional Alternating Direction Explicit Procedure -- Grid Points and Time Levels for the Forward and Reverse Sweep	103
F-1	Relative Permeability Curves for Oil and Water	138
F-2	Water Viscosity Versus Temperature	139
F-3	Oil Viscosity Versus Temperature	140

INTRODUCTION

The future demands for oil appear favorable. The economic law of supply and demand requires that an increased demand should be followed by an increased supply to stabilize the unfavorable balance. The problem of increasing the supply may be met in two ways by the petroleum industry; 1) increase exploration, and/or 2) improve recovery techniques of the existing oil reserves.

As a result of alternative number 2), cold-water flooding became a widely accepted secondary recovery mechanism. There still remained, however, a class of oil reservoirs which would not readily yield to production by primary or secondary recovery techniques -- the heavy oil (high viscosity oil) reservoirs. Thermal recovery processes were thus initiated as a means for production of heavy oils with extremely temperature sensitive viscosities.

One such thermal recovery process, steam injection, has aroused great interest and controversy. Successes and failures of pilot projects involving steam stimulation of an oil reservoir have been witnessed. The reasons for success or failure in most cases have not been published but it is believed that economics plays

a major role in most of the failures.

The development of a mathematical model to simulate an oil reservoir undergoing a thermal change appears desirable. A brief review of available analytical solutions (1-15) for describing some aspects of the thermal recovery process for hot-water, hot-gas, and steam injection was presented by Flock et al (16). They point out that the analytical solutions generally describe either the thermal phenomena or the fluid flow aspect but not both. Davidson (17) states that the thermal and hydrodynamical phenomena should be analyzed simultaneously and that a rigorous treatment of heat losses is essential to developing a realistic model.

To describe completely and accurately a reservoir system subjected to thermal stimulation, a complex mathematical model is required. The resulting partial differential equations which satisfy the model and treat the thermal and hydrodynamical phenomena and the heat losses in a rigorous manner are highly nonlinear.

Realizing the shortcomings of attempting to solve this complex problem by analytical means, recent authors(16,17,18,19,20,21,22) have adopted numerical techniques which are solvable on high-speed computers.

This work was done with the following purposes in mind:

1) to review the theory and literature on steam injection (three-phase flow) and hot-water injection (two-phase flow).

2) to develop a mathematical model which would describe the hydrodynamic and thermal behavior of a reservoir system, and to discuss the validity of the major assumptions incorporated in the formulation of the model.

3) to discuss various numerical techniques available to solve the aforementioned mathematical model, and thence to employ a method which will make the solution to the problem both economically and computationally feasible.

4) to compare the results of this work with the results of previous authors, if possible.

THEORY AND LITERATURE REVIEW -- STEAM INJECTION

THREE PHASE FLOW

The intermittent steam injection process has been studied by a number of authors(16,17,18,21,22,23,24). The physical interpretation of this process would be as follows. Steam is injected into the entire reservoir thickness or into a highly permeable fracture for a specified time (one to three weeks (17)). Upon completion of the injection period the well is shut in and the pressure begins to decline. This soaking period (two days to one week(17)), allows time for the transfer of the latent heat of vaporization of the steam to the reservoir system. Having allowed sufficient time for the steam to condense, the well is open for production. Production continues until the producing rate which prevailed prior to steam stimulation is approached, or until the economics of production becomes unfavorable. At this time steam injection is resumed and the cycle of injection, soaking and production is repeated; hence the name intermittent or cyclic steam injection.

A complete analysis of the cyclic steam injection process therefore involves three stages:

1) a description of the injection process,

2) a description of the soaking (condensation) period, and
3) a back-production analysis.

Davidson (17) developed a general model to describe the intermittent injection of steam. Realizing the uneconomic possibilities of attempting to solve the general model, he reduced the model to three one-dimensional simplified cases. A portion of this work was presented by Davidson et al (24). Flock et al (16) presented a two-dimensional mathematical model to describe the injection period and a second model for describing the back-flow period. The soaking period was not described.

The need for an approximate but computationally feasible model inspired other workers (18,21,22) to develop models describing the "huff-and-puff", "push-pull" or cyclic steam injection process.

The major practical drawback to this thermal recovery mechanism is the loss of production during the injection and soaking periods. A steam displacement process whereby injection and production of fluids occur

in different wells rather than in the same well as is the case for cyclic steam injection would eliminate the necessity for lost production time which is inherent in the cyclic process. The models developed for the injection period of the cyclic process should describe equally well the steam displacement process.

Figure 1 is a representation of the physical system. The mathematical model to be developed departs from the true physical problem due to the underlying assumptions incorporated. The limitations of the model will therefore depend on the seriousness of these assumptions and on any restrictions imposed on the solution to the problem due to the use of numerical techniques.

The major assumptions incorporated in the steam injection mathematical model are listed.

1. A mathematical procedure may be used to describe the physical system.
2. The reservoir is assumed to be homogeneous with respect to porosity and absolute

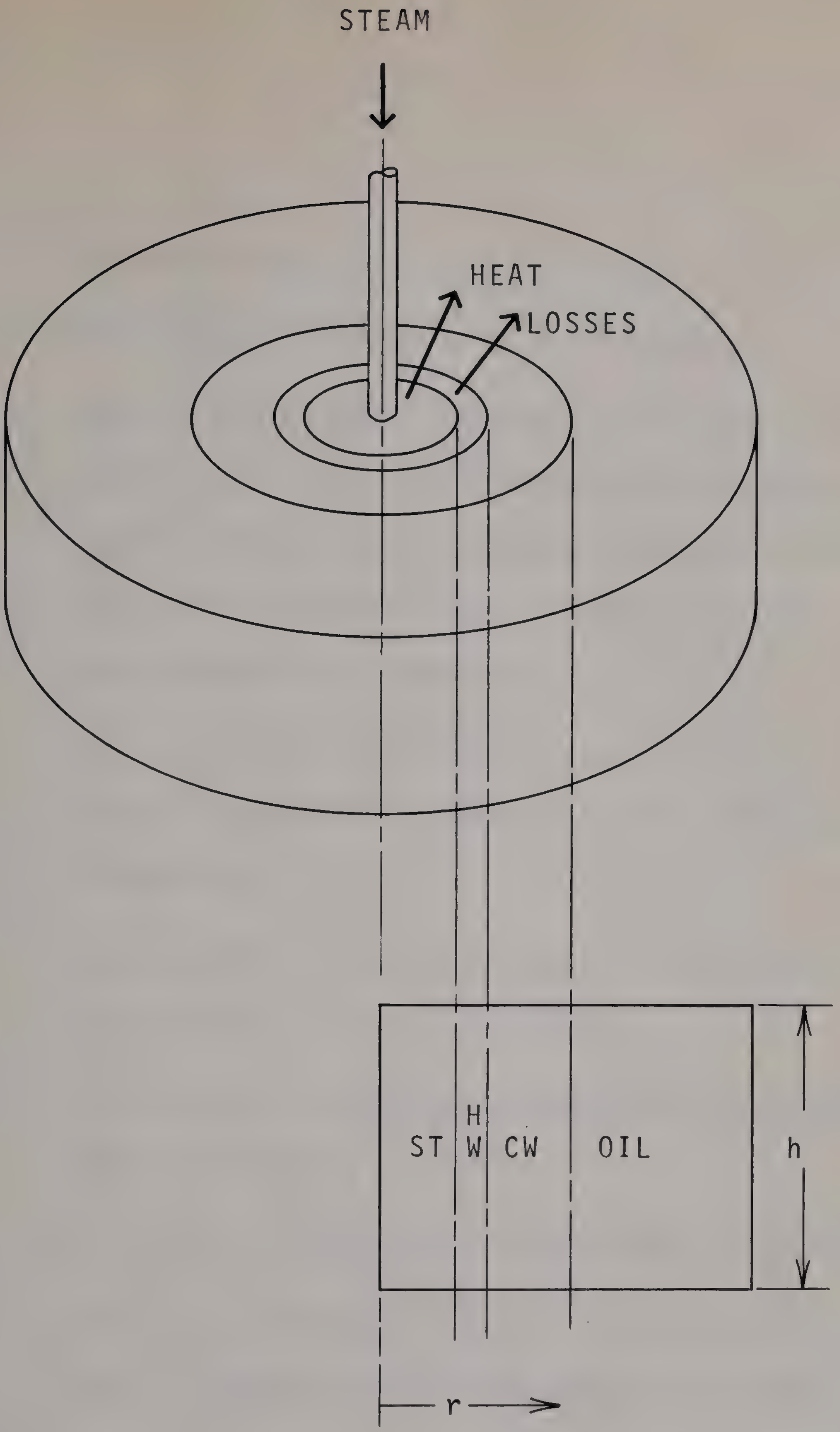


DIAGRAM OF THE PHYSICAL SYSTEM--STEAM DISPLACEMENT

FIGURE 1

permeability.

3. The reservoir thickness is a constant.
4. All fluids exhibit newtonian behavior in the reservoir. That is, a linear relationship exists between the shearing stress and the velocity gradient, with viscosity as the proportionality constant.
5. The fluid velocity may be represented by Darcy's Law and may be applied to each phase independently.
6. Arising from the use of Darcy's Law, fluid interaction is not considered.
7. Darcy's Law is valid for nonisothermal flow (see Appendix D).
8. The relative permeability concept is valid(25).
9. Capillary pressure effects are negligible. This is equivalent to saying the pressure in each phase is the same and is equal to the system pressure.

10. There is no free gas or gas in solution in the reservoir.
11. Oil production increases are due to viscosity reduction, thermal expansion and fluid displacement. This assumes that there are no volatile components in the reservoir fluid and therefore steam distillation is not considered as a driving mechanism.
12. The wellbore heat losses are negligible.
13. The use of an effective thermal conductivity in the energy equation is acceptable (see Appendix D).
14. The reservoir rock is considered incompressible.

The problem resolves itself into describing the simultaneous flow of three fluids, oil, water, and steam, under the influence of a pressure gradient due to the injection of steam and a thermal gradient as a result of the steam injection temperature. A detailed derivation of the steam injection model is presented in Appendix A.

The continuity equation for a compressible fluid may be written as follows:

$$\left|\begin{array}{c}\text{Rate of change}\\ \text{of mass}\\ \text{per unit volume}\end{array}\right| = \left|\begin{array}{c}\text{Net mass flux by}\\ \text{bulk transport}\\ \text{per unit volume}\end{array}\right| + \left|\begin{array}{c}\text{Rate of mass in-}\\ \text{crease due to}\\ \text{vaporization per}\\ \text{unit volume}\end{array}\right|$$

$$+ \left|\begin{array}{c}\text{Net mass flux by}\\ \text{molecular}\\ \text{diffusion per}\\ \text{unit volume}\end{array}\right|$$

or in mathematical notation, neglecting mass flux by molecular diffusion, the statement becomes

$$-\phi \frac{\partial}{\partial t}(\rho_i S_i) = \nabla \circ \rho_i \vec{v}_i + V_i (T,P) \tag{1}$$

where the subscript (i) represents a particular phase. (see nomenclature for symbol definitions)

Darcy's Law neglecting gravity effects (horizontal flow) and capillary forces may be written for each phase.

$$\vec{q}_i = -\frac{k_i}{\mu_i}\frac{\partial P}{\partial r} = \frac{\vec{Q}_i}{2\pi r h} = \vec{v}_i \tag{2}$$

For a radial system, the divergence $(\nabla \cdot \vec{A})$ in cylindrical coordinates for horizontal, one-dimensional flow is represented by (26)

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (rA_r) \tag{3}$$

Combining equations (3) and (2) with equation (1), the hydrodynamical equations are obtained for each phase.

Oil Phase

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\rho_o k_o}{\mu_o} \frac{\partial P}{\partial r} \right) - \phi \frac{\partial}{\partial t} (\rho_o S_o) = 0 \tag{4}$$

Water Phase

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\rho_w k_w}{\mu_w} \frac{\partial P}{\partial r} \right) - V_w(T_o,P) - \phi \frac{\partial}{\partial t} (\rho_w S_w) = 0 \tag{5}$$

Steam Phase

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\rho_{st} k_{st}}{\mu_{st}} \frac{\partial P}{\partial r} \right) + V_w(T_o,P) - \phi \frac{\partial}{\partial t} (\rho_{st} S_{st}) = 0 \tag{6}$$

From material balance considerations,

$$S_o + S_w + S_{st} = 1.0 \qquad (7)$$

The thermal energy equation as stated by Bird, Stewart and Lightfoot (27) is,

Rate of gain of internal energy per unit volume	=	Rate of internal energy input by conduction per unit volume	+	Reversible rate of internal energy increase per unit volume by compression
	+	irreversible rate of internal energy increase per unit volume by viscous dissipation	+	net gain due to condensation per unit volume

or in mathematical terms, the statement may be written as,

$$\rho \frac{DU}{Dt} = - \nabla \cdot \vec{q}_t - p(\nabla \cdot \vec{v}) - \tau : \nabla \vec{v} + V(T_o, P) \qquad (8)$$

The internal energy U is not a convenient property for working petroleum engineering problems. Manipulation of the basic thermodynamic functions enables the transformation of equation (8) to an equation involving the system temperature and heat capacity rather than the internal energy.

$$\rho C_v \frac{DT_o}{Dt} = -\nabla\cdot\vec{q}_t - T_o\left(\frac{\partial P}{\partial T}\right)_v \nabla\cdot\vec{v}_{st} - \tau:\nabla\vec{v} + V(T_o,P) \quad (9)$$

The heat losses to the overburden and underburden may be described mathematically in one dimension by Fouriers Law.

$$\frac{\partial^2 T_1}{\partial z^2} = \frac{1}{\alpha_{OB}} \frac{\partial T_1}{\partial t} \quad (10)$$

The theoretical model thus consists of five non-linear partial differential equations with variable coefficients. The independent variables are r, t, and z while the dependent variables are P, T_o, T_1, S_o, S_w, and S_{st}. A realistic set of boundary and initial conditions for a constant mass injection rate of water (G_w) and steam (G_{st}) would be:

Initial Conditions

$P(r,0) = P_i$ $\qquad T_o(r,0) = T_i$

$T_1(r,0) = T_i$ $\qquad S_o(r,0) = S_{oi}$

$S_w(r,0) = S_{wi}$ $\qquad S_{st}(r,0) = 0.0$

Boundary Conditions

$$P\,(r_e,t) = P_i \qquad T_o(r_w,t) = T_{st}$$

$$\left.\frac{\partial T_o}{\partial r}\right|_{r=r_e} = 0 \qquad \left.\frac{\partial T_o}{\partial z}\right|_{z=0} = 0$$

$$\left.\frac{\partial T_1}{\partial r}\right|_{r=r_e} = 0 \qquad \left.\frac{\partial S_o}{\partial r}\right|_{r=r_e} = 0$$

$$\left.\frac{\partial S_w}{\partial r}\right|_{r=r_e} = 0 \qquad \left.\frac{\partial S_{st}}{\partial r}\right|_{r=r_e} = 0$$

$$S_o(r_w,t) = 0 \qquad G_{st} = \left(\frac{-\rho_{st}k_{st}}{\mu_{st}}\frac{\partial P}{\partial r}\right)_{r=r_w}$$

$$G_w = \left(\frac{-\rho_w k_w}{\mu_w}\frac{\partial P}{\partial r}\right)_{r=r_w} \qquad k_e\left.\frac{\partial T_o}{\partial Z}\right|_{z=\frac{h}{2}} = k_{OB}\left.\frac{\partial T_1}{\partial z}\right|_{z=\frac{h}{2}}$$

$$T_o\Big|_{z=\frac{h}{2}} = T_1\Big|_{z=\frac{h}{2}}$$

To complete the model the fluid densities must be presented in appropriate forms as functions of pressure and temperature.

Equations (4) through (10) with appropriate boundary and initial conditions and equations of state comprise the mathematical model to describe a steam displacement process. A generalized mathematical model describing the thermal recovery of oil in linear systems was presented by Gottfried(20). Six coupled, non-linear, partial differential equations with variable coefficients were required to describe the three-phase fluid flow model which included conduction-convection heat transfer, reaction between oxygen and oil and aqueous phase change. The analysis was applied to the detailed simulation of a forward-combustion laboratory tube experiment. Two to three hours of computer time were required to simulate twenty-four hours of real time.

The similarity of Gottfried's model and the one presented in this work would suggest that similar computer time might be required to solve the problem and this was deemed uneconomical.

THEORY AND LITERATURE REVIEW - HOT-WATER INJECTION

TWO PHASE FLOW

Although the complex mathematical steam displacement model developed in the previous section and described fully in Appendix A will not be solved in this work, it will serve as a useful basis for simpler models of hot-water injection or hot, noncondensible gas injection.

In treating hot-water injection or hot, noncondensible gas injection it is also desirable to have an accurate description of the thermal and hydrodynamical phenomena occurring. The analytical solutions reviewed by Flock et al (16) as mentioned earlier, did not satisfy both requirements.

Fournier(19) proposed a model for prediction of oil recovery by hot-water injection incorporating the effects of viscosity ratio reduction, thermal expansion and heat losses to the overburden and underburden for a radial system. He treats the fluids as incompressible.

A representation of the physical system for the present work is shown in Figure 2. Figure 3(b) shows that the system is an "open-flow" boundary problem.

DIAGRAM OF THE PHYSICAL SYSTEM -- HOT WATER INJECTION

FIGURE 2

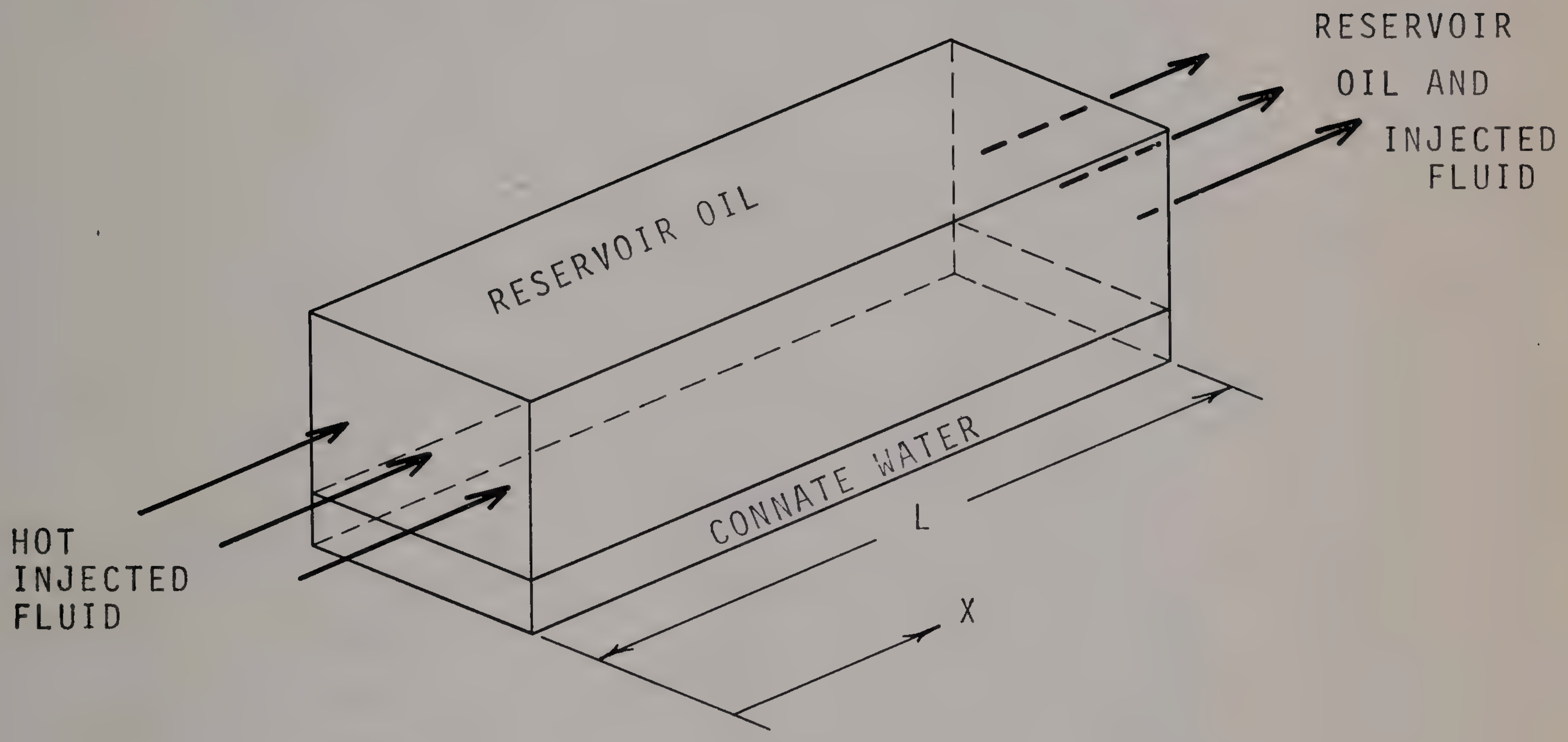
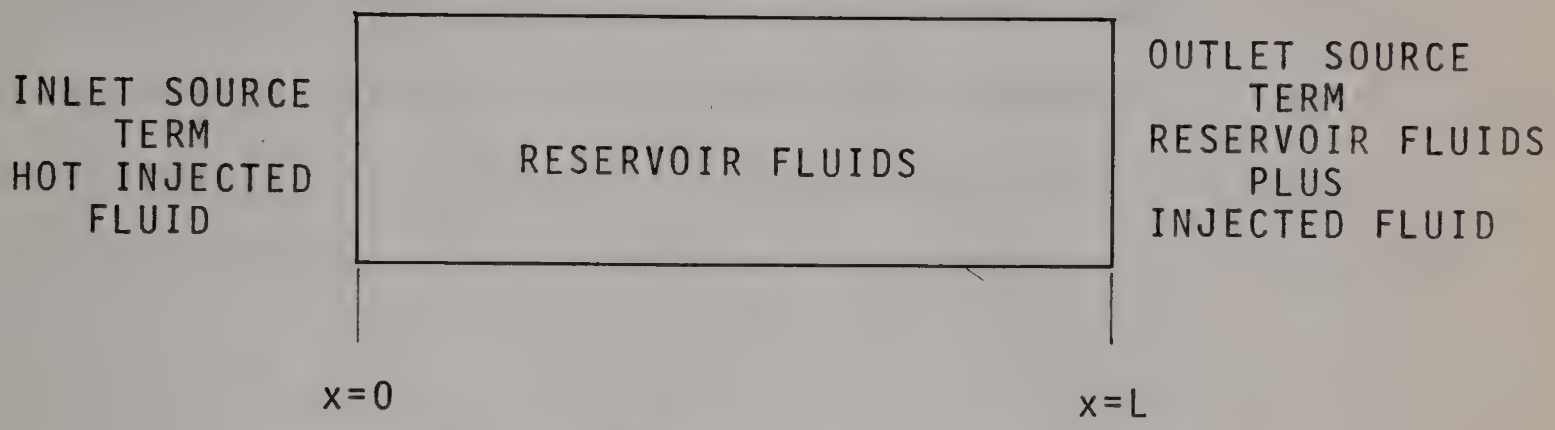


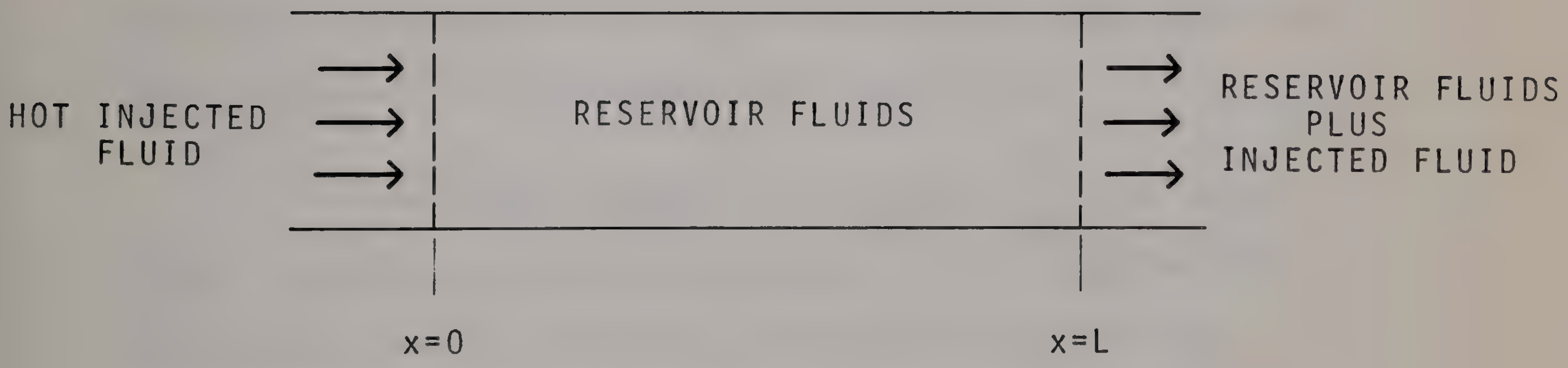
DIAGRAM OF "NO-FLOW" AND "OPEN-FLOW" BOUNDARIES FOR RESERVOIR MODELS

FIGURE 3



"NO-FLOW" BOUNDARY CASE

(a)



"OPEN-FLOW" BOUNDARY CASE

(b)

Figure 3(a) shows another possible model, the "no-flow" boundary problem. The mathematical development of the two problems differs but it will be interesting to note that the finite difference approximations to the partial differential equations are identical as shown in Appendix D.

This study will deal with the "open-flow" boundary problem as illustrated in Figure 2 and Figure 3(b) and therefore only its derivation will be considered in this section. Appendix D shows the derivation of the "no-flow" boundary problem and a comparison between the partial differential equations and the finite difference formulations for both models is illustrated.

The major assumptions employed in the "open-flow" boundary problem derivation are the same as those presented for the steam displacement model with the additional assumption that the heat losses to the overburden and underburden may be neglected.

In considering hot-water injection or hot, noncondensible gas injection the problem involves a

description of the simultaneous flow of two fluids, reservoir oil and injected fluid, under the influence of a pressure and thermal gradient.

For a linear hot-water injection process the combined continuity-Darcy flow equations (4) and (5) assume the form; (see Appendix B for the complete derivation)

Oil Phase

$$\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\left[\frac{\partial}{\partial t}(S_o) + C_o S_o \frac{\partial P}{\partial t}\right] \qquad (11)$$

Water Phase

$$\frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\left[\frac{\partial}{\partial t}(-S_o) + C_w(1-S_o)\frac{\partial P}{\partial t}\right] \qquad (12)$$

and equation (7), from material balance considerations, reduces to,

$$S_o + S_w = 1.0 \qquad (13)$$

The thermal energy equation (9) neglecting the viscous dissipation terms (assuming high velocity gradients do not exist in the reservoir) and the reversible interchange of mechanical and thermal energy due to compression or expansion (important for gases only) becomes:

$$\rho C_v \frac{DT_o}{Dt} = -\nabla \cdot \vec{q}_t \tag{14}$$

With the appropriate substitutions the thermal energy equation assumes its final form;

$$\rho C_p \frac{\partial T}{\partial t} = CC1 \left\{ \left[\rho_o C_{p_o} \frac{k_o}{\mu_o} + \rho_w C_{p_w} \frac{k_w}{\mu_w} \right] \frac{\partial P}{\partial x} \right\} \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(k_e \frac{\partial T}{\partial x} \right) \tag{15}$$

$$\rho C_p = [\rho_w C_{p_w} (1-S_o) + \rho_o C_{p_o} S_o]\, \phi + \rho_r C_{p_r} \, (1-\phi) \tag{16}$$

The dependent variables are P, T, S_o, and S_w and the independent variables are x and t.

Finally, specification of the initial and boundary conditions to be imposed on the mathematical model for a constant mass injection rate of water (G_w) completes the problem.

Initial Conditions

$S_o\ (x,0) = S_{oi}$ $\qquad$ $S_w\ (x,0) = S_{w_i}$

$P\ (x,0) = P_i$ $\qquad$ $T\ (x,0) = T_i$

Boundary Conditions

$S_o\ (0,t) = 0.0$ $\qquad$ $S_w\ (0,t) = 1.0$

$T\ (0,t) = T_{INJ}$ $\qquad$ $G_w(0,t) = -CC1\ \frac{\rho_w k_w}{\mu_w} \left.\frac{\partial P}{\partial x}\right|_{x=0} = \text{CONSTANT}$

$G_o\ (0,t) = 0.0$ $\qquad$ $\left.\frac{\partial S_o}{\partial x}\right|_{x=L} = 0.0$

$\left.\frac{\partial S_w}{\partial x}\right|_{x=L} = 0.0$ $\qquad$ $\left.\frac{\partial T}{\partial x}\right|_{x=L} = 0.0$

$P(L,t) = P_i$

Equations (11) through (15) with the necessary boundary and initial conditions and appropriate relations for the fluid densities as functions of pressure mathematically describe

the hot-water injection model neglecting heat losses to the overburden and underburden.

DISCUSSION OF POSSIBLE METHODS OF SOLUTION OF THE HOT-WATER INJECTION MODEL

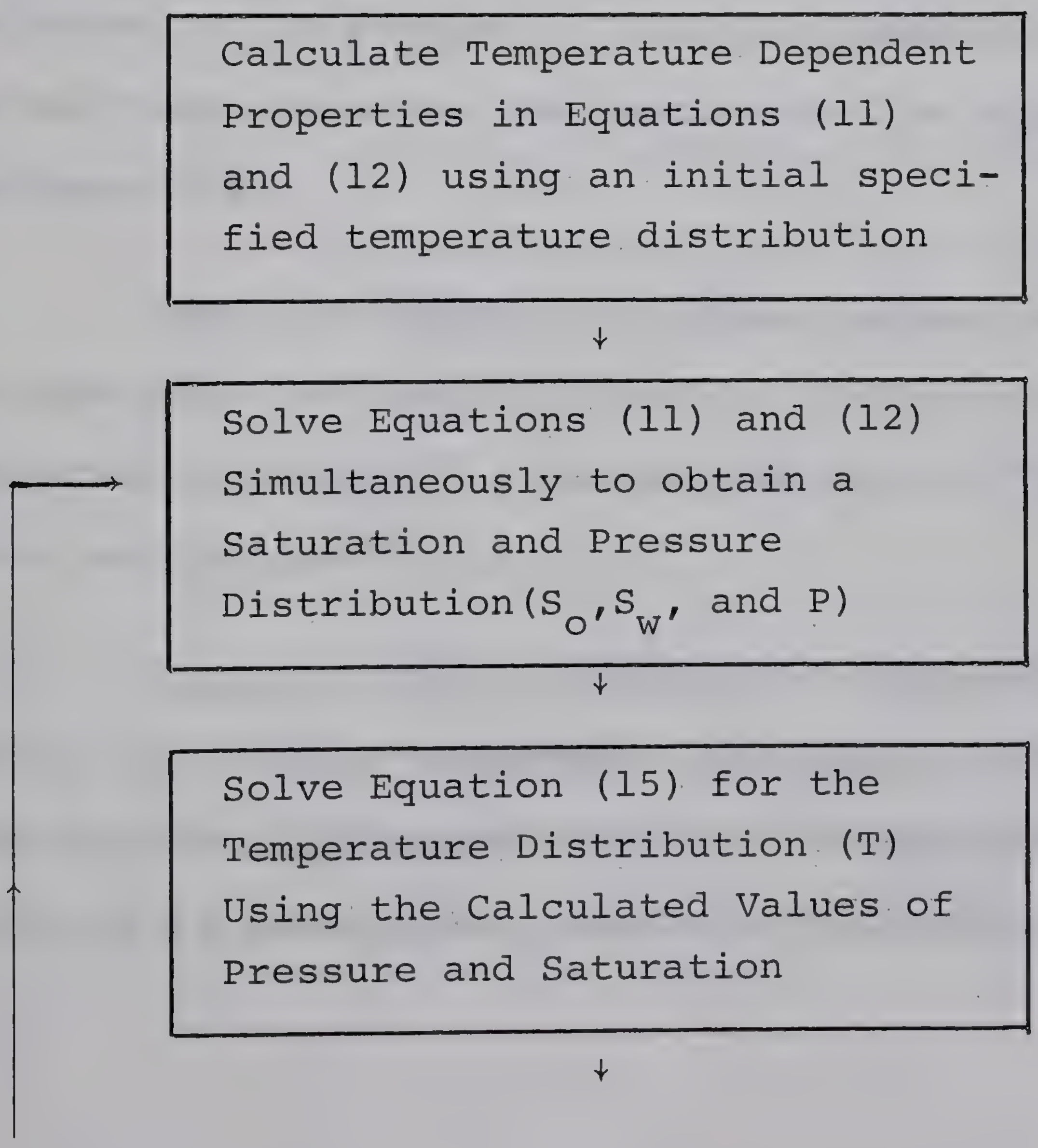
The solution of the hot-water injection problem will require a numerical technique. As stated previously, any restrictions imposed on the solution to the problem due to the use of numerical techniques will result in a limitation of the model.

Assuming a solution to the mathematical problem exists and will satisfy the physical problem, a method of solution must be prescribed.

The simultaneous solution of the continuity-flow equations (equations (11) and (12)) with the thermal energy equation (15) would be desirable. An attempt at this solution was unsuccessful, however, no conclusions were drawn from the failure to obtain a valid solution. Rather than pursue the problem of trying to solve three non-linear partial differential equations with variable coefficients simultaneously, a second method was prescribed.

Since the continuity-flow equations are both parabolic, their solution should be possible by some

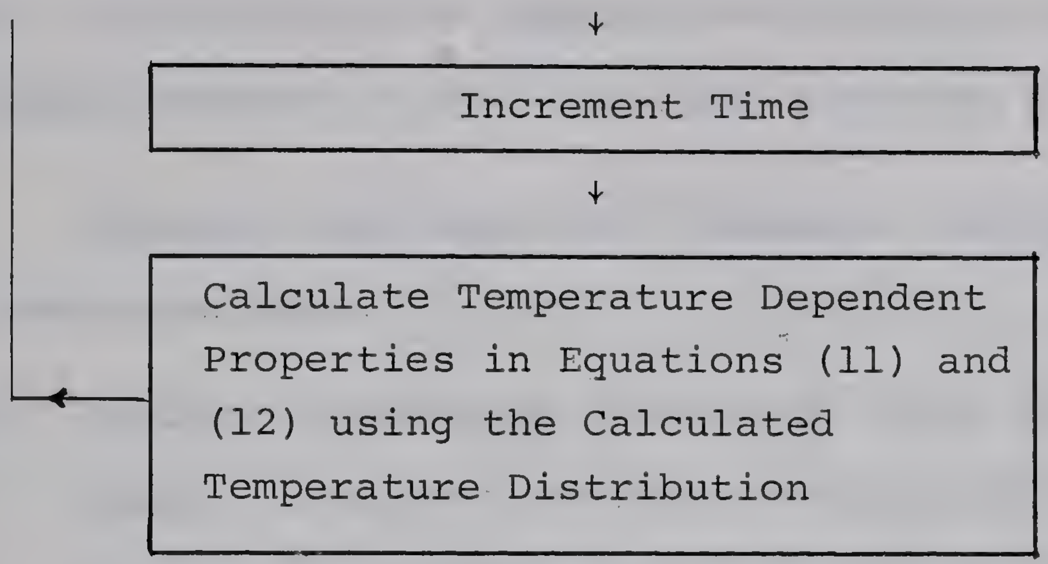
existing numerical technique. Their solution would yield a pressure and saturation distribution over a given time interval. This would require that all temperature dependent properties would have to be calculated using a known value of temperature. The calculated pressure and saturation distributions could be used in the thermal energy equation to solve for a new temperature distribution at the end of the time step.



Calculate Temperature Dependent Properties in Equations (11) and (12) using an initial specified temperature distribution

Solve Equations (11) and (12) Simultaneously to obtain a Saturation and Pressure Distribution (S_o, S_w, and P)

Solve Equation (15) for the Temperature Distribution (T) Using the Calculated Values of Pressure and Saturation


Increment Time
Calculate Temperature Dependent Properties in Equations (11) and (12) using the Calculated Temperature Distribution

Assuming this prescribed procedure will yield a solution to the problem, a suitable numerical technique must be chosen to solve the continuity-flow equations simultaneously.

The first attempts to obtain solutions of the two-phase flow problems by numerical techniques could be attributed to Muskat(28), Terwilleger et al(29)and West, Garvin and Sheldon(30).

Douglas, Blair, and Wagner(31)determined the behavior of a linear waterflood employing a modified Crank-Nicolson difference equation to approximate their second order, quasilinear, parabolic differential

equation. The difference equation was implicit and second order correct in both time and distance.

Peaceman and Rachford(32) discuss two types of difference equations.

1) explicit difference equations, which are simple to solve, but which require an uneconomically large number of time steps of limited size. The forward-difference explicit (FDE) procedure fits this classification.

2) implicit difference equations, which do not limit the time step but which require at each time step the solution by iteration of large sets of simultaneous equations. The backward-difference implicit (BDI) procedure fits this classification.

The authors present an alternating-direction implicit procedure (ADIP) which requires the line-by-line solution of small sets of simultaneous equations that can be solved by a direct, non-iterative method and is stable

for any size time steps. Iterative techniques are also discussed.

Douglas, Peaceman and Rachford(33) employed ADIP to solve a two-dimensional, two-phase, incompressible flow model which included the effects of gravity and capillarity in the formulation. Blair and Peaceman(34) extended the work of Douglas, Peaceman, and Rachford to include the effect of compressibility of the fluids. They successfully predicted downdip displacement of oil by gas in a laboratory model by numerical solution of the two-dimensional, two-phase, fluid flow equations using ADIP.

Gottfried, Guilinger, and Snyder(35) present two numerical methods (explicit and implicit) for solving the equations for one-dimensional multiphase flow in porous media. Variable physical properties are included in the formulation although not used for the problems solved. Gravity and capillary forces were neglected. The explicit and implicit procedures were essentially of about the same order accuracy, with the implicit method requiring 15 to 20% more machine

time but allowing use of a larger time step for a given mesh spacing.

Quon et al(36)discuss five numerical methods for handling parabolic differential equations - the forward-difference explicit (FDE), the backward-difference implicit (BDI), the Crank-Nicolson implicit (CNI), the alternating-direction implicit procedure (ADIP) and the alternating-direction explicit procedure (ADEP). Their main suggestion was that ADIP and ADEP computational models would describe equally well a physical or mathematical problem but that ADEP would offer greater computational efficiency. A second demonstration of the use of ADEP(37)shows that it offers an advantage over the FDE procedure from a stability viewpoint, thus permitting larger time steps; and it offers an advantage over ADIP from a computational point of view since only one non-linear algebraic equation had to be solved at a time rather than a set of equations, representing a row or column of grid points.

Coats and Terhune(38)compared the accuracy and computing speed of ADEP and ADIP in numerical

solution of reservoir fluid flow problems by analyzing published example applications. They concluded that ADIP yields significantly greater accuracy but requires about 60 percent more computing time than ADEP.

Carter(39) offers further discussion to the Coates and Terhune paper. His experience with ADIP and ADEP indicates: (1) that ADEP extends the range of conditions for which explicit methods may be used in practice to obtain gas flow solutions; (2) ADIP is a method of more general utility than ADEP; and (3) the additional storage and computer time required by ADIP over ADEP should normally be of little concern with present day computers.

This study is concerned with a one-dimensional two-phase flow problem and involves the simultaneous solution of two non-linear parabolic partial differential equations with variable coefficients. The alternating-direction explicit procedure (ADEP) has not been applied to a one-dimensional problem of this type as far as the author knows and hence the

solution to this problem was carried out by ADEP along with the Newton-Raphson iterative technique.

Crank-Nicolson finite difference equations and the square root method for inverting a matrix, extended to non-symmetric matrices, were employed in the solution of the thermal energy equation.

Appendix C shows the detailed discretization and final formulation of the finite difference equations which were solved on an IBM 7040 digital computer.

DISCUSSION OF CONVERGENCE AND STABILITY OF THE COMPUTER PROGRAM

In the application of numerical techniques convergence may be defined as the approach of the approximate numerical solution to the exact solution as the number of increments employed increases and in the limit approaches infinity as the time step size approaches zero. This criteria is sufficient if an exact solution is known, however, in many cases as in this work the exact solution is not known and some other criteria for convergence must be established. For the problem considered in this work convergence was assumed when the results obtained from using two different time step sizes (one time step size being one half the value of the other) compared within the "acceptable limits" of the problem. "Acceptable limits" are self defined and could be termed as the tolerance allowed between the two sets of answers obtained when the computer program is executed for the two different time step sizes.

The program, although stable for large time steps, was not satisfactorily convergent. Also, for large time steps an oscillation in the pressure

parameter was witnessed for small values of real time. This oscillation dampened out with increasing values of real times.

To obtain a satisfactory convergent solution it was necessary to employ very small time steps, in fact, so small that it became uneconomical in that one hour of computer time was required to simulate two hours of real time.

In order to remedy the situation, that is to make the ratio of simulated real time to computer time as large as possible, the time step size must be increased or the number of grid points must be reduced. This is contrary to the first definition of convergence which stated that the grid size spacing should be decreased. It is thus evident that the grid size spacing may be increased at the expense of decreasing the accuracy of the solution. A decision must be made as to whether or not the accuracy of the results may be given some flexibility so that an economical solution may be obtained. The desired harmony between the accuracy of the results and the

size of the allowable time step will yield an economical and computationally feasible model.

The critical time step concept for the Forward-Difference Explicit (FDE) method is not too useful for non-linear problems of this type as the fluid properties and hence the coefficients of the equations are not constant which suggests that the critical time step is also not a constant.

The work presented in reference (35) served as a useful guide in determining an approximate value for grid size spacing and time step size and for the general "trend" of the results to be expected for two-phase isothermal flow.

DISCUSSION OF RESULTS

If an isothermal fluid injection problem is being considered the actual amount of oil displacement and hence oil recovery during the injection process will depend upon the relative ease with which the two fluids (oil and water) can move through the porous media. The readiness with which a fluid flows increases with its saturation in the porous sand. During the initial stages of displacement the oil saturation in the model is high and the displacing water saturation is low and therefore from a relative permeability standpoint the oil will flow readily in comparison with the water. As displacement continues the permeability of the sand to oil decreases while the permeability of the sand to water increases (Figure F-1, Appendix F) until eventually large volumes of water will effect removal of only a small amount of additional oil.

When a reservoir undergoes thermal stimulation from steam injection, it is believed that viscosity reduction, thermal expansion and steam distillation are the primary mechanisms responsible for additional

oil recovery. For the hot-water injection model considered in this work only the viscosity reduction mechanism was considered and both the oil and water viscosities were treated as temperature sensitive. The function definitions for water and oil viscosities as given in Appendix F are:

$$\mu_w(T) = \left[\frac{1776-T}{26.5(T)-89}\right] \tag{17}$$

$$\mu_o(T) = \mu_o^* \, (60)^{\nu} \, T^{-\nu} \tag{18}$$

where the value of (ν) determines the sensitivity of the oil viscosity to temperature changes as shown in Figure (F-3) in Appendix F.

Thus, for the isothermal problems considered in this work oil production will depend on the displacement process only, while for the nonisothermal problems the oil production will depend on the displacement process and the viscosity reduction mechanism.

Since the reservoir oil has no ability to expel itself, the oil recovery will depend upon the availability of the displacing water and on its accumulation during the displacement process. The availability of the displacing water may be specified but to determine the accumulation of water and hence the efficiency of displacement it is necessary to know the pressure, saturation and temperature conditions existing in the model at various times and positions along the length of the model. The results presented in this work are the pressure, saturation and temperature distributions necessary to predict the oil recovery for an isothermal or nonisothermal water injection problem and are shown in the figures at the end of this section. These figures were plotted from the computer output presented in Appendix G. Although it was not possible to have a check on the results or the model, the curves are representative of classical or typical shapes found in the literature(35,40).

Figures 4, 5, and 6 show the pressure distribution along the total length of the linear model

for indicated values of grid size spacing (Δx) and time step size (Δt) for one particular value of real time and for isothermal and nonisothermal injection conditions. Three values of the oil viscosity sensitivity factor ($\surd$) were used; 0.56, 1.12, and 1.50 as shown respectively on Figures 4,5, and 6.

Figure 7 is a plot of the saturation conditions in the model which correspond with the pressure distributions shown on Figures 4, 5, and 6. These four curves are presented to show the error induced in the results if the grid size spacing is increased to allow use of larger time steps and less grid points so as to reduce the total computing time requirements. The curves represented by the smallest grid size spacing and time step size are used as reference curves from which deviations of the other curves may be compared.

It is evident that the choice of grid size spacing ($\Delta x = 0.300$ feet) is not the best one from the point of view of accuracy but it was believed that the combined harmony of accuracy and total computing time required to solve the problem makes this choice

the most economical and computationally feasible with the existing computing facilities. Guided by this criteria, the computer program was run for the following conditions for both isothermal and nonisothermal injection.

TABLE I

√	Δx(feet)	Δt(hours)
0.56	0.300	0.015
1.12	0.300	0.020
1.50	0.300	0.015

Gottfried et al(35) show a family of pressure curves for a two-phase flow problem restricted to isothermal and incompressible systems. A sketch of their pressure profiles is shown in Figure 23 of this work. The following points should be observed.

(1) The linear portion of the curves represents the pressure distribution ahead of the "flood front". For compressible systems this part of the curve should not be linear.

(2) The non-linear parts of the curves represent the pressure distribution behind the "flood front", that is, the two phase flow region.

(3) As real time increases, the "flood front" advances and the pressure drop through the model decreases in the region behind the "flood front" since the resistance to flow decreases.

Figures 8, 13, and 18 in this work represent the pressure distributions along the model length for isothermal and nonisothermal injection conditions. The curves exhibit similar "trends" to the results of Gottfried et al (35). The following observations may be made.

(1) Although the system (oil and water) was treated as compressible the curves still remained essentially linear ahead of the "flood front". This is attributable to the low injection rate (low pressure) and the very slight compressibility of liquids at low pressures. For high injection pressure and for gas flow it is believed that

compressibility will be important and hence the mathematical model was made general.

(2) The nonisothermal pressure gradients are lower than the isothermal pressure gradients for the same value of real time at large values of real time. The thermal stimulation supposedly decreases the flow resistance and thus the pressure drop decreases in the thermally stimulated regions.

(3) The curves do not converge to one common linear pressure distribution line ahead of the "flood front" for small values of real time. The error is caused by the choice of a large grid spacing and time step which was discussed earlier. For the nonisothermal system conduction ahead of the "flood front" stimulates the region as discussed in point (2) and the pressure drop should not be along the same line if conduction effects are important. The errors induced, however, appear to be tolerable.

Figures 9, 14, and 19 represent the saturation distributions and Figures 11, 16, and 21 show the cumulative oil production histories for isothermal and nonisothermal flow conditions corresponding with the pressure distributions shown in Figures 8, 13, and 18.

TABLE II

Time (Hrs.)	Oil Recovery (Fraction of Oil in Place)	Maximum Frontal Advance (Feet)	√
36	0.194 (0.214)*	2.25(2.55)	0.56
36	0.174 (0.159)	1.95(1.95)	1.12
36	0.194 (0.184)	2.25(2.25)	1.50

* The bracketed values are for nonisothermal conditions and the unbracketed values are for isothermal conditions.

It would seem reasonable that the greater the sensitivity of the oil viscosity to temperature changes the faster the "flood front" would advance and the greater the oil recovery should be. Figure 9 illustrates a faster advance of the "flood front" for the nonisothermal system and Figure 11 illustrates the greater recovery for

the nonisothermal system. To be more specific, looking at Table II for a value of oil sensitivity factor (√=0.56), the frontal advance for the nonisothermal system is 2.55 feet while the advance for the isothermal system is only 2.25 feet. Similarly the nonisothermal system depletes 21.4 percent of the original oil in place as compared with only 19.4 percent for the isothermal problem. Since the curves on Figure 19 and 21 are for a higher sensitive oil viscosity (√=1.50) they should show an even more pronounced frontal advance and greater recoveries. Observation of the curves shows the reverse situation occurring at large values of real time. Looking at Table II for a value of √=1.50 the maximum frontal advance of 2.25 feet is the same for both systems while the recovery of the nonisothermal problem (18.4%) is less than the recovery of the isothermal problem (19.4%). This would seem to indicate that the use of the large grid spacing is not acceptable or that it induces larger and larger errors as the fluid properties become more and more non-linear. Figures 14 and 16 show respectively the saturation distribution and the cumulative oil production for the

case where the oil sensitivity factor (√) equals 1.12. Again undesirable results are obtained as seen from the curves and from Table II. The maximum frontal advance is the same for both isothermal and nonisothermal flow (1.95 feet) and the production under isothermal conditions (17.4%) is greater than the production under nonisothermal conditions (15.9%). For this case the combined effects of large grid spacing, large time step and highly sensitive oil viscosity produced the errors in the results. For the isothermal systems the results should be identical for all problems but looking at Table II it may be seen that for √=0.56 and √=1.50 this condition is satisfied but for √=1.12 the results are not identical. This may be explained by inspecting Table I. For √=0.56 and √=1.50 the grid spacing and time step sizes (Δt) are the same, For √=1.12 the grid spacing is the same but the time step is larger. The following deductions may be stated. For a small value of the oil sensitivity factor (√= 0.56) the use of the large grid spacing (Δx=0.300 feet) and a time step size of Δt=0.015 hours is acceptable.

As the oil sensitivity factor increases the use of large grid spacings and large time steps yields undesirable results.

It will be interesting to note that the isothermal systems have only pressure as the driving force while the nonisothermal systems have pressure and temperature driving forces. It was mentioned earlier that the pressure gradients for the nonisothermal systems were smaller than for the corresponding isothermal systems and that as time increased the differences became more pronounced. In actual fact, for the conditions of this problem the water will not breakthrough the oulet end of the system for the nonisothermal systems. Figure 8 may be used to provide an explanation. It is observable that the nonisothermal pressure gradients are approaching the initial pressure faster than the isothermal gradients. Eventually the pressure distribution will equal the initial pressure distribution and there will be no pressure driving force and no flow. It happens that for this problem the pressures equalized before breakthrough and the only

solution would be to increase the injection rate and thus the injection pressure. The computer output in Appendix G (Computer Output I) for the nonisothermal system and for a real time of forty-five hours shows the pressure equalization. Since this phenomena was not anticipated there was no lower bound placed on the pressure and the results show the inlet injection pressures being lower than the outlet pressure and hence reverse flow occurs. A lower bound on the pressure such that it cannot go below the value at the outlet end of the model should have been incorporated in the program.

Finally, Figures 12, 17, and 22 show the cumulative material balances for each flowing phase. The material balance should equal 1.0 and deviations from this value indicate possible errors in the method employed or in the choice of the grid spacing and time step. The curves are self explanatory.

In summary, the alternating direction explicit procedure (ADEP), written for one-dimension, was

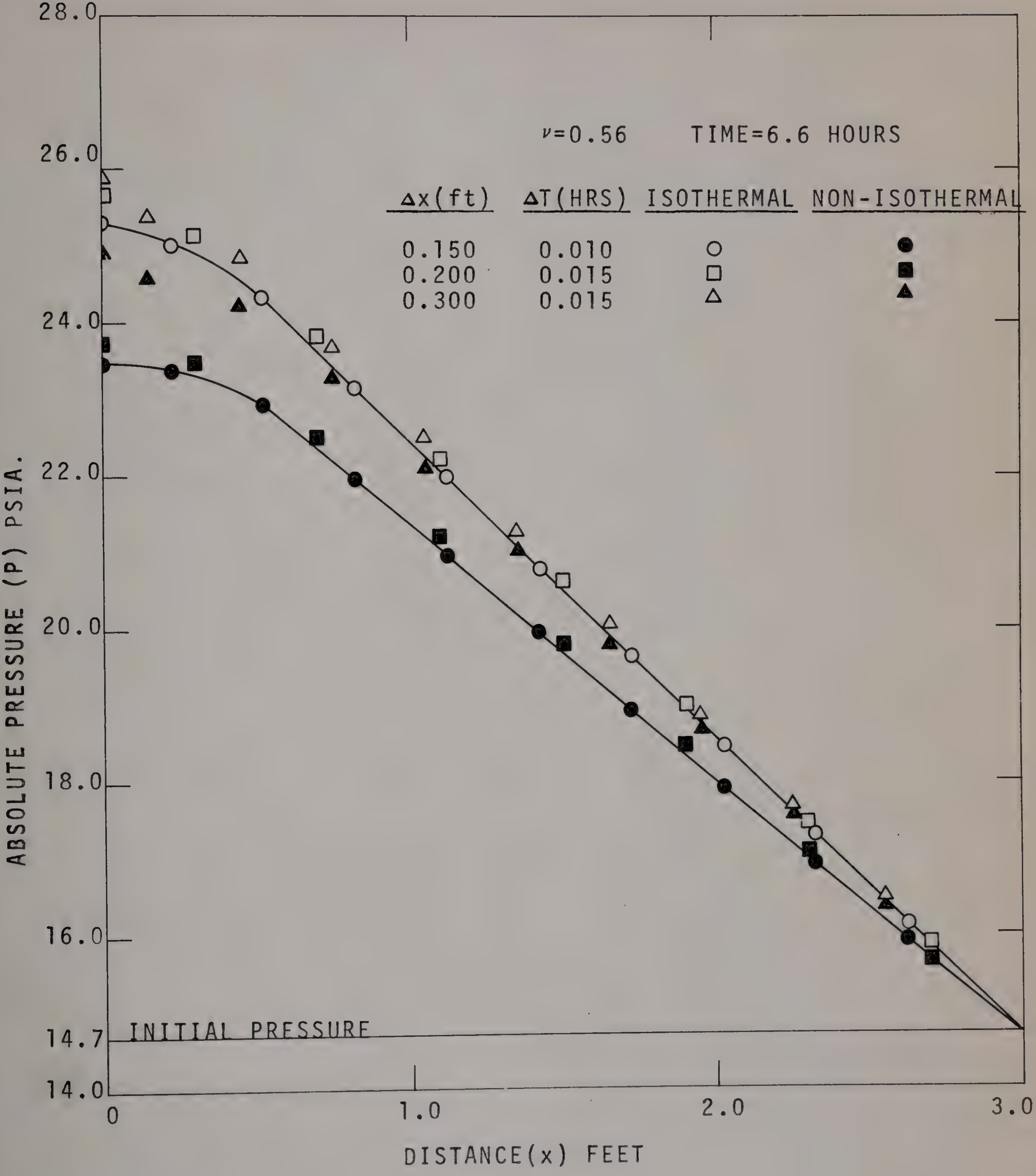
successfully applied to solve an isothermal, compressible, two phase flow problem. The thermal aspect of the problem was treated separately and the heat losses to the overburden and underburden were neglected. The author is the first to admit that the developed model is not highly sophisticated from the point of view of simulating an actual thermal model but the experience and insight gained from this study leads the author to believe that the combined thermal and hydrodynamical aspect as well as a rigorous treatment of heat losses to the overburden and underburden could be considered simultaneously employing the same approach used to solve the simple isothermal two phase flow problem.

Although the method appeared to be unconditionally stable, convergence problems were encountered. The choice of grid size spacing and time step size for non-linear problems of this type must be made on a trial-and-error basis or from previous experience (which the author cannot claim). The choice made in this work was such that an economical and computationally feasible model could be

developed. The economics of required computing time should not impose any restrictions for future workers as the IBM-7040 computing facilities at the University of Alberta are presently being replaced by an IBM-360-67 computer which is approximately six times faster. Smaller grid size spacing and smaller time step sizes may be used making the results more accurate and allowing calculation of the highly non-linear problems possible.

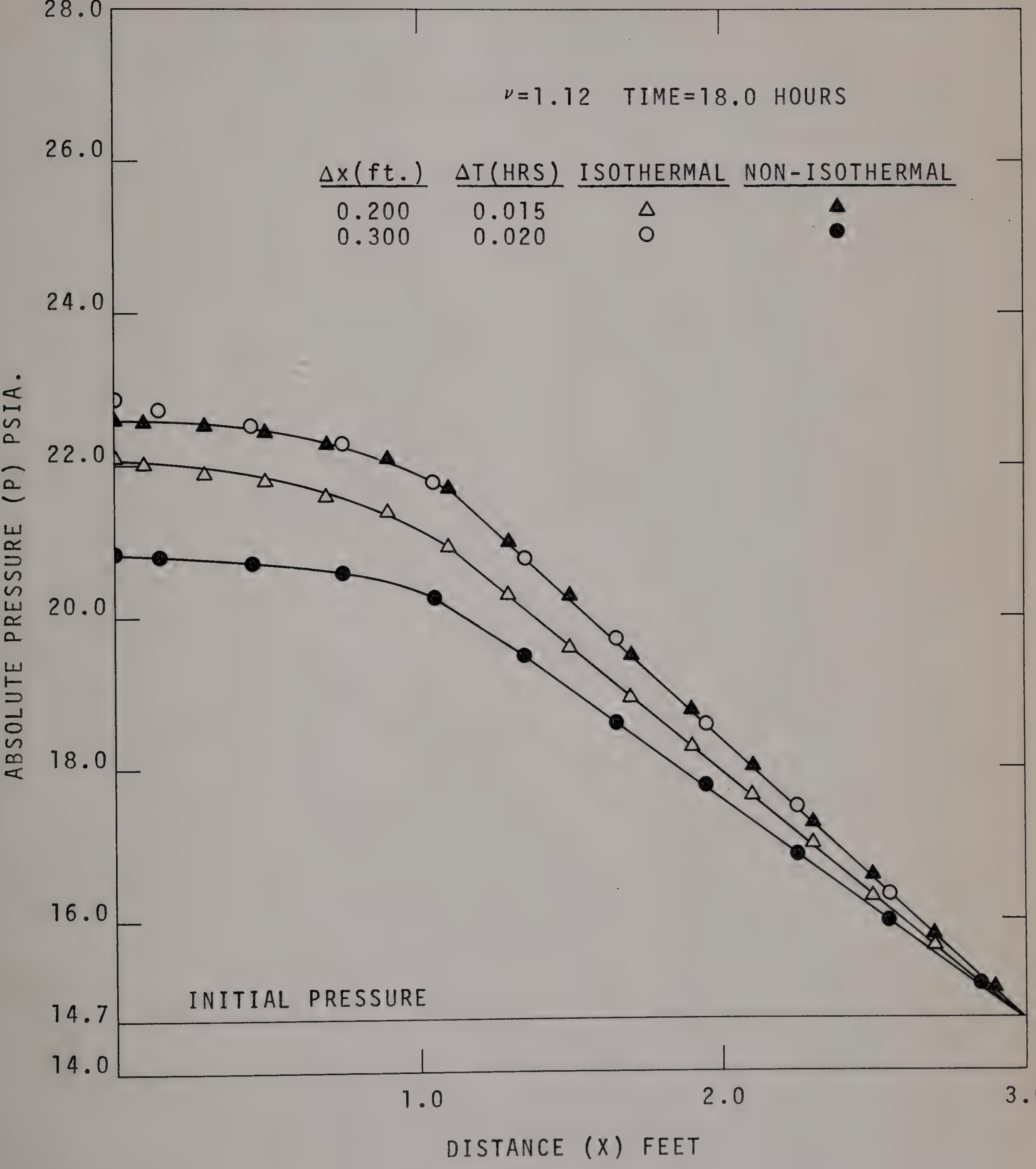
PRESSURE DISTRIBUTIONS

FIGURE 4



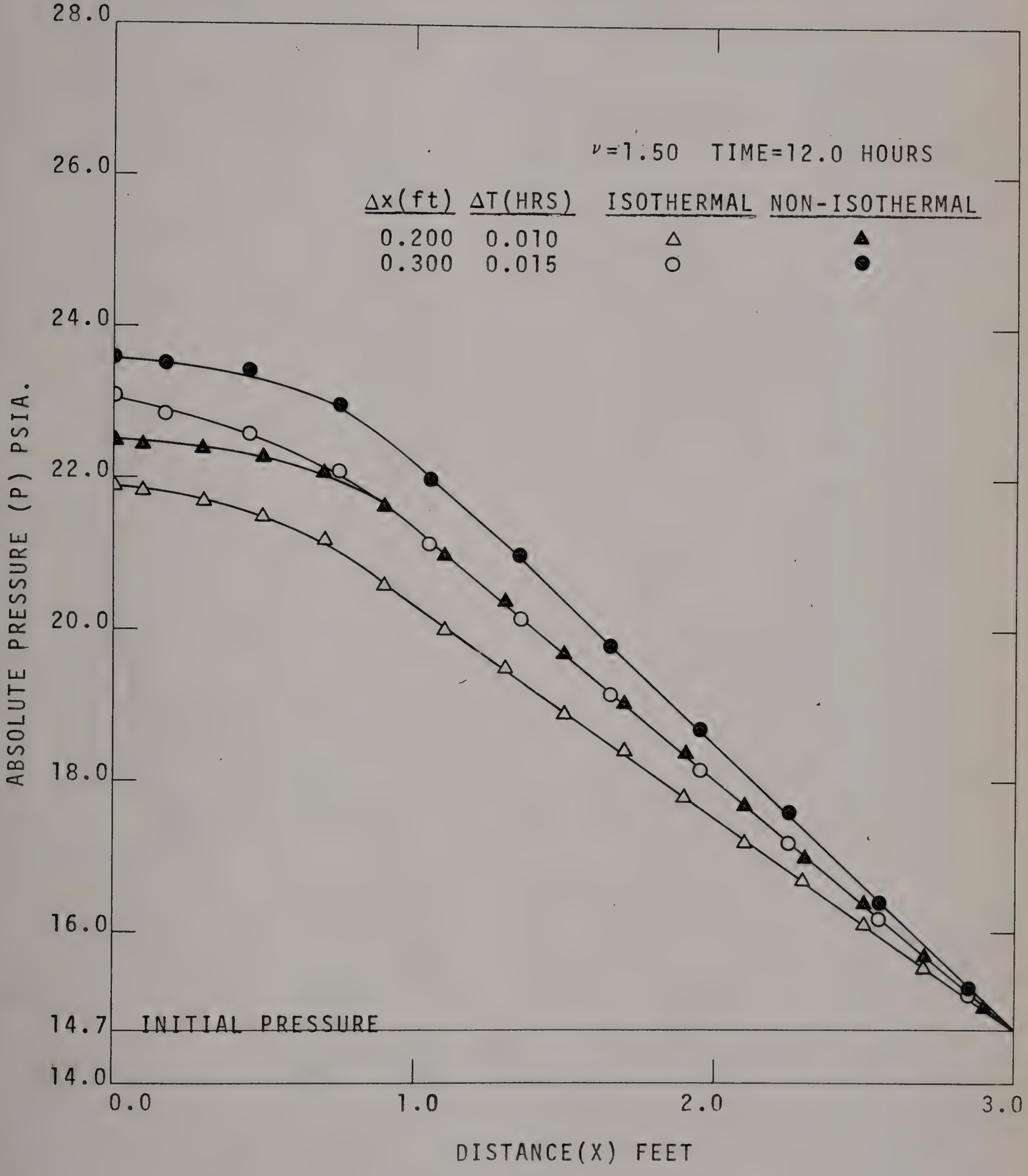
PRESSURE DISTRIBUTIONS

FIGURE 5



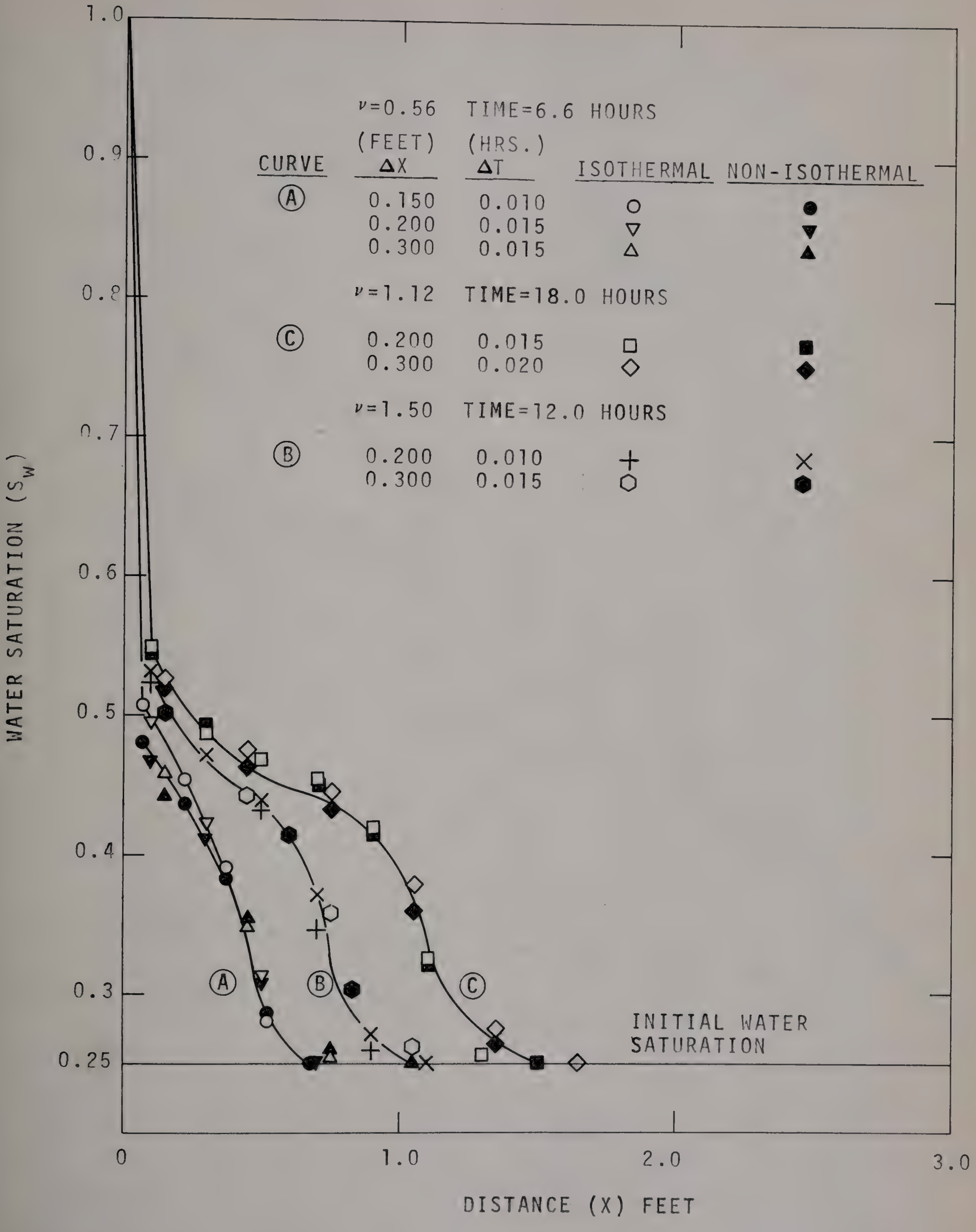
PRESSURE DISTRIBUTIONS

FIGURE 6



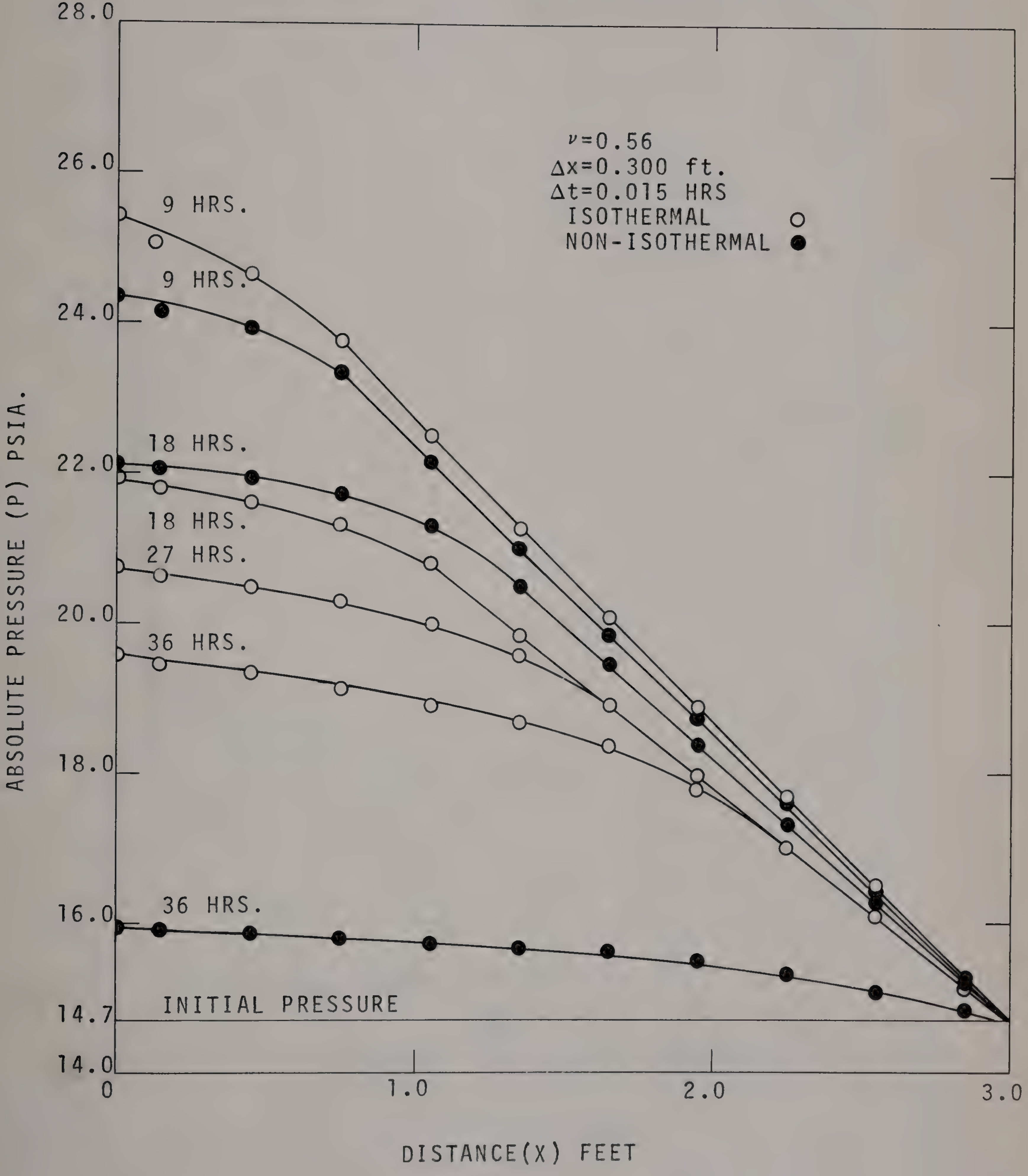
SATURATION DISTRIBUTIONS

FIGURE 7



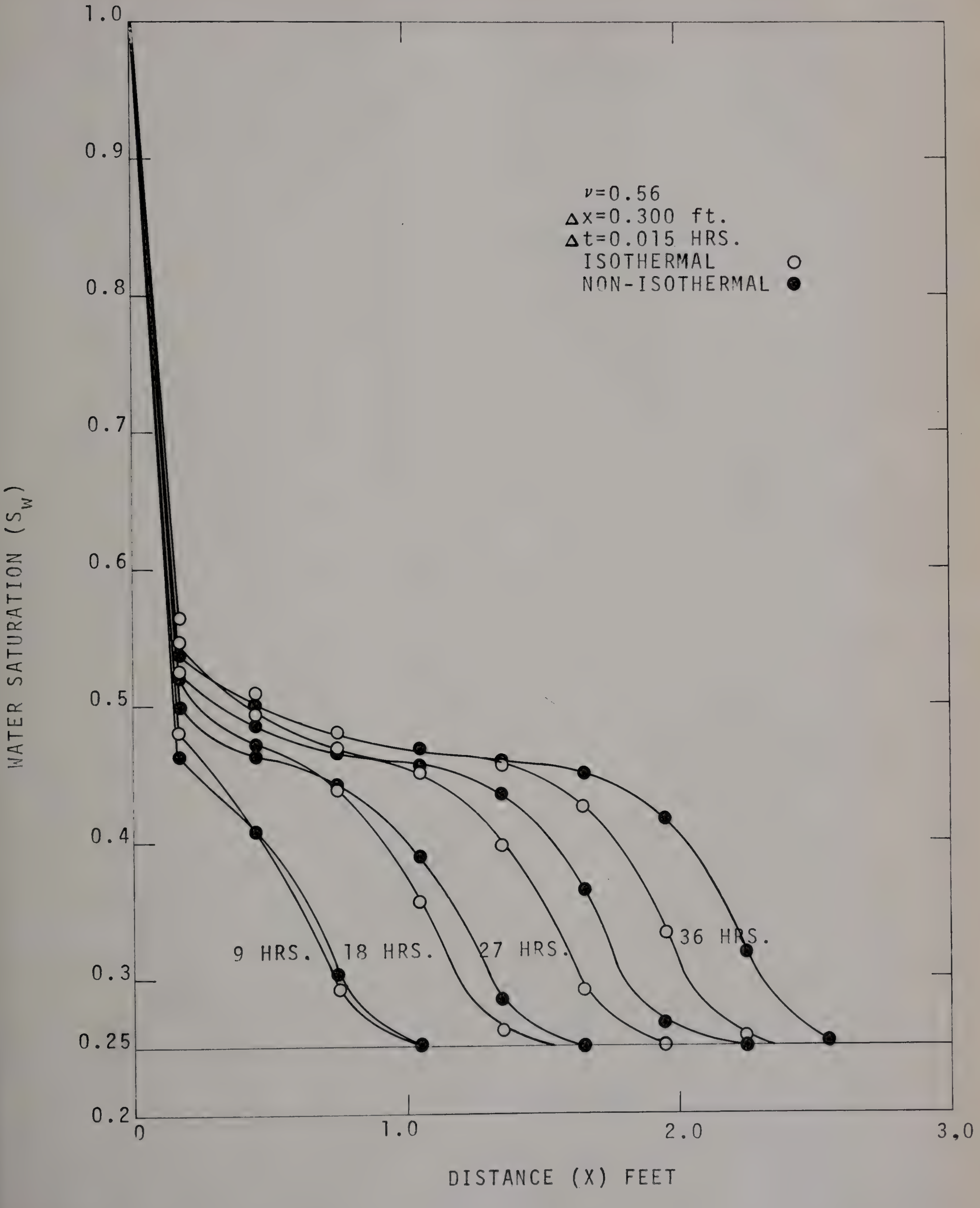
PRESSURE DISTRIBUTIONS

FIGURE 8



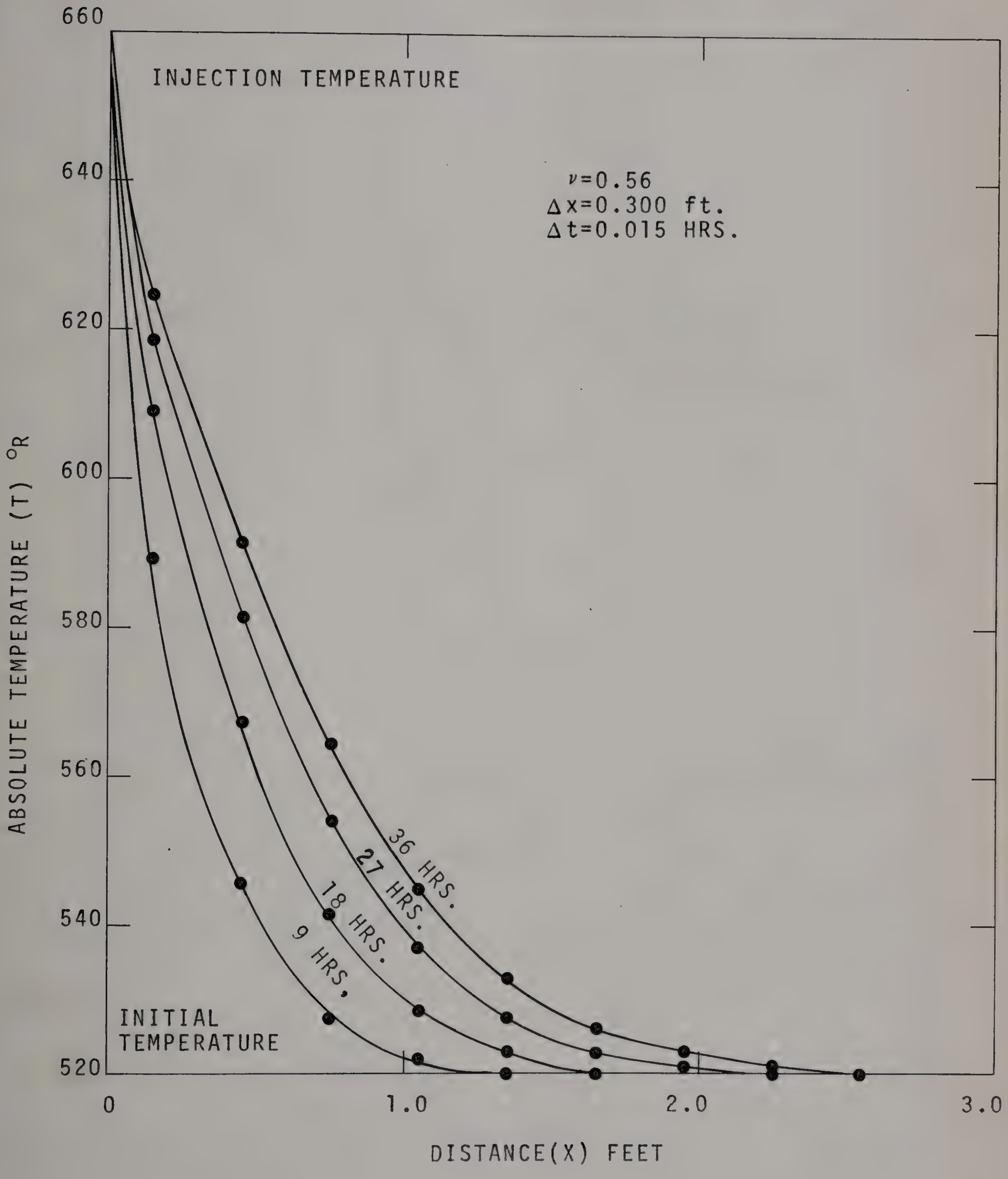
SATURATION DISTRIBUTIONS

FIGURE 9



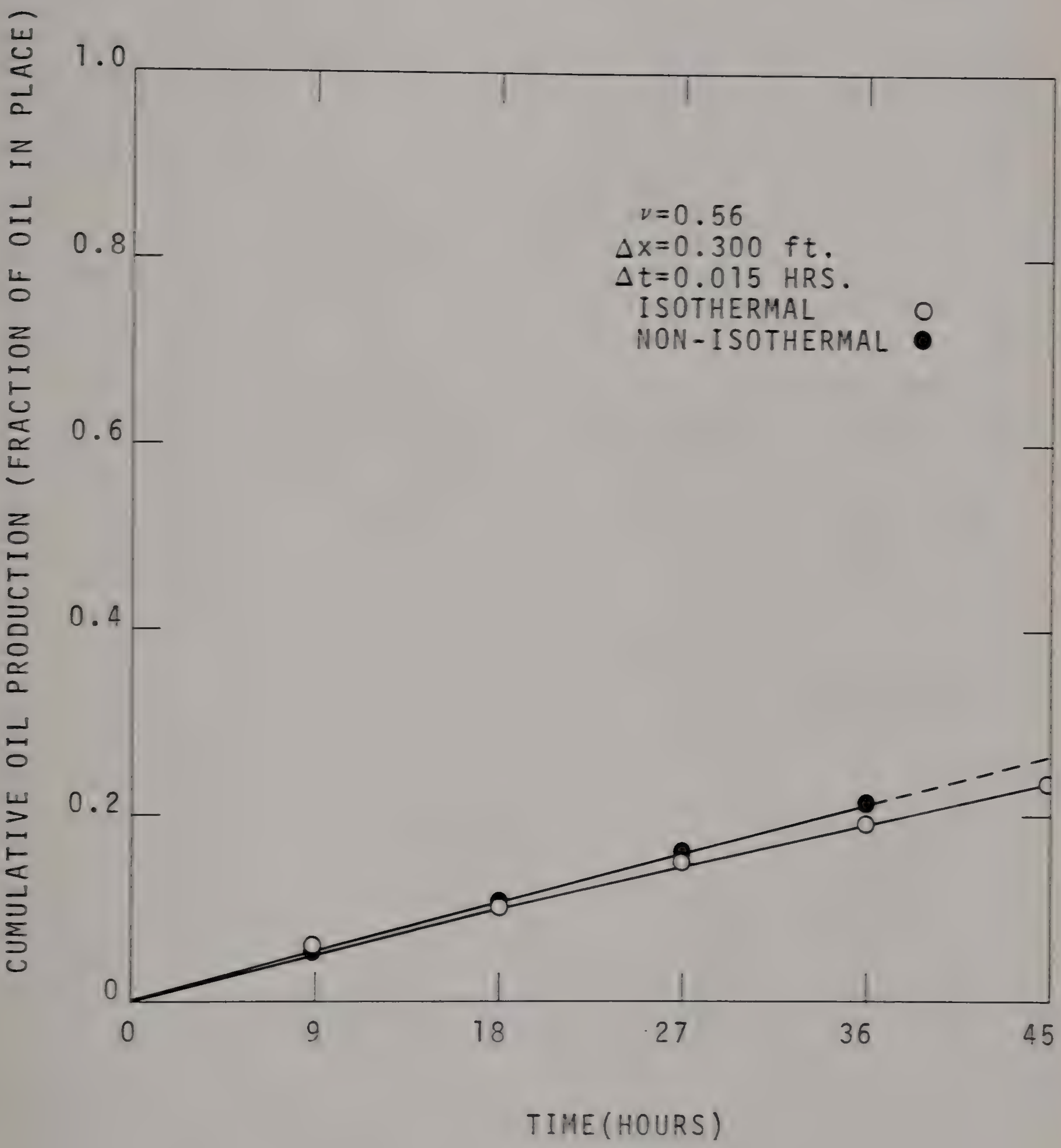
TEMPERATURE DISTRIBUTIONS

FIGURE 10



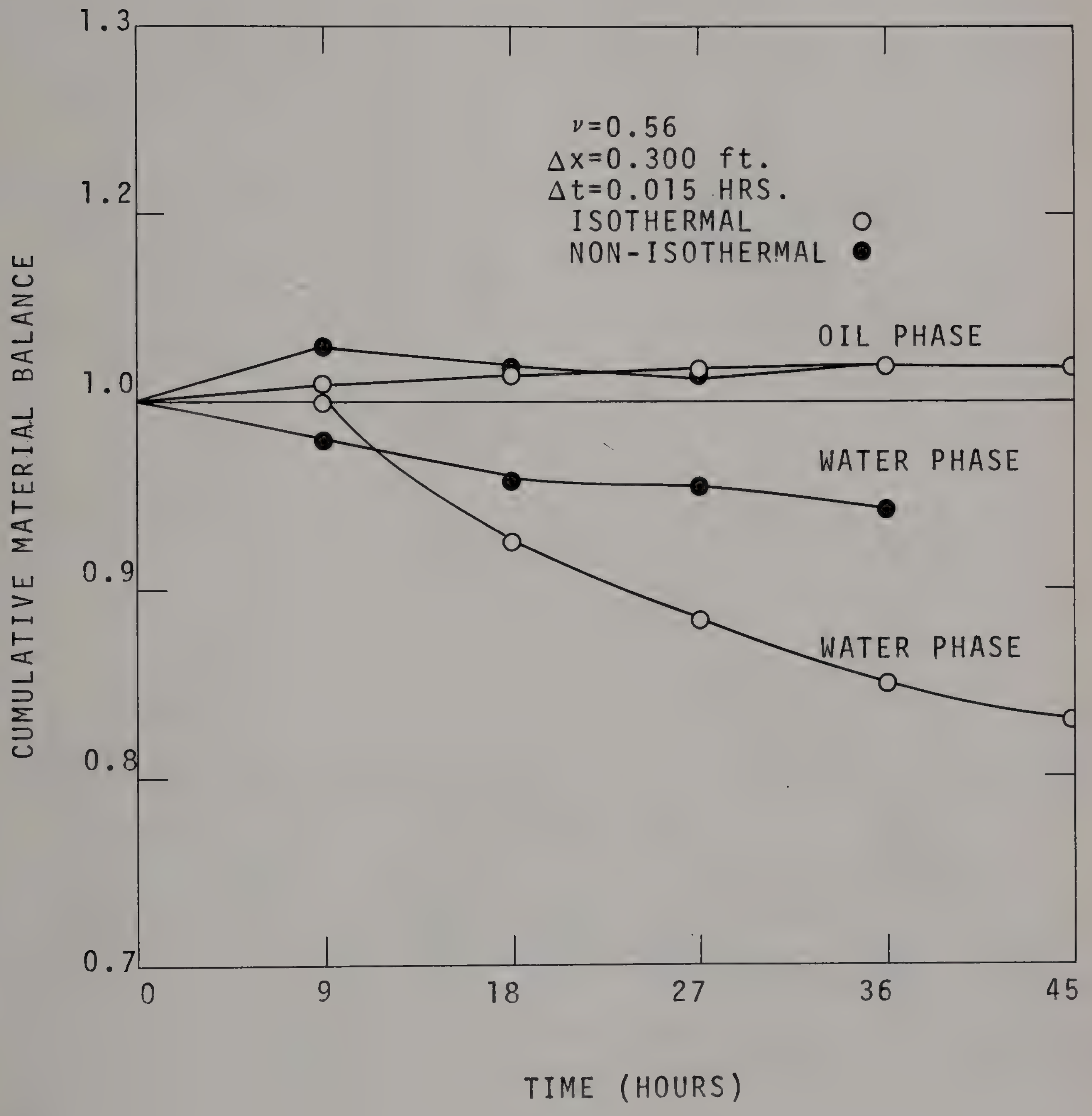
CUMULATIVE OIL PRODUCTION HISTORY

FIGURE 11



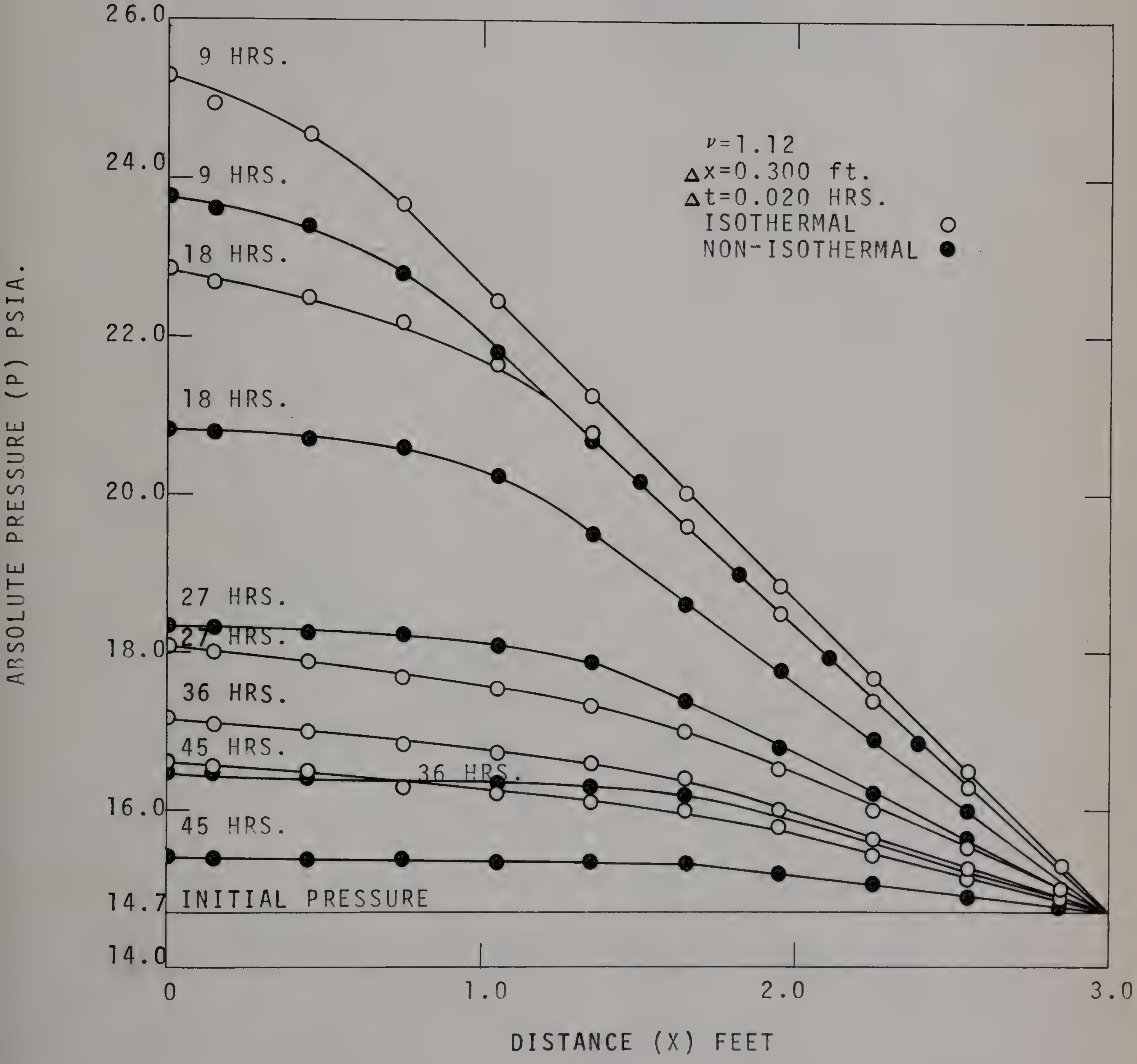
CUMULATIVE MATERIAL BALANCES

FIGURE 12



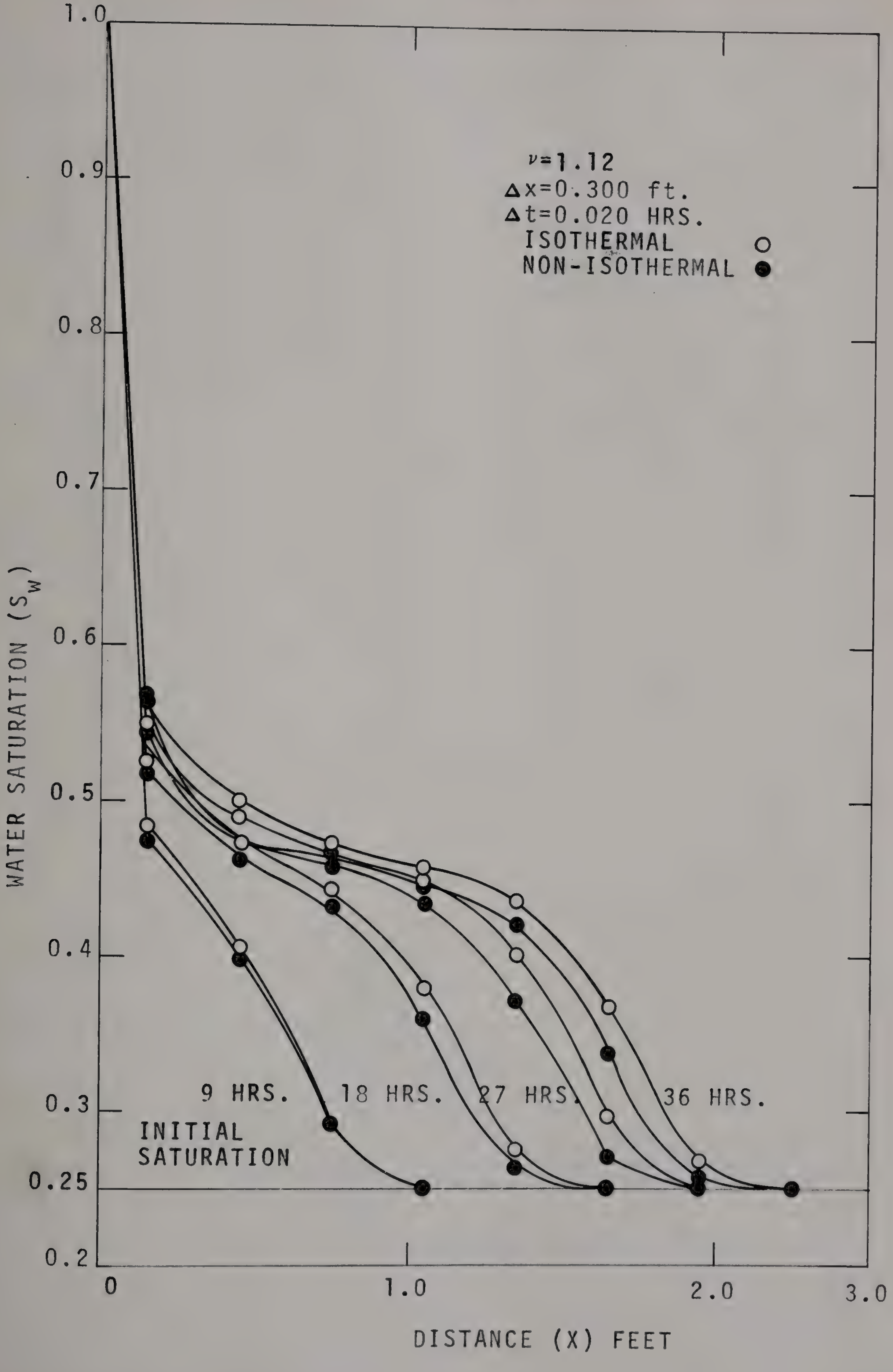
PRESSURE DISTRIBUTIONS

FIGURE 13



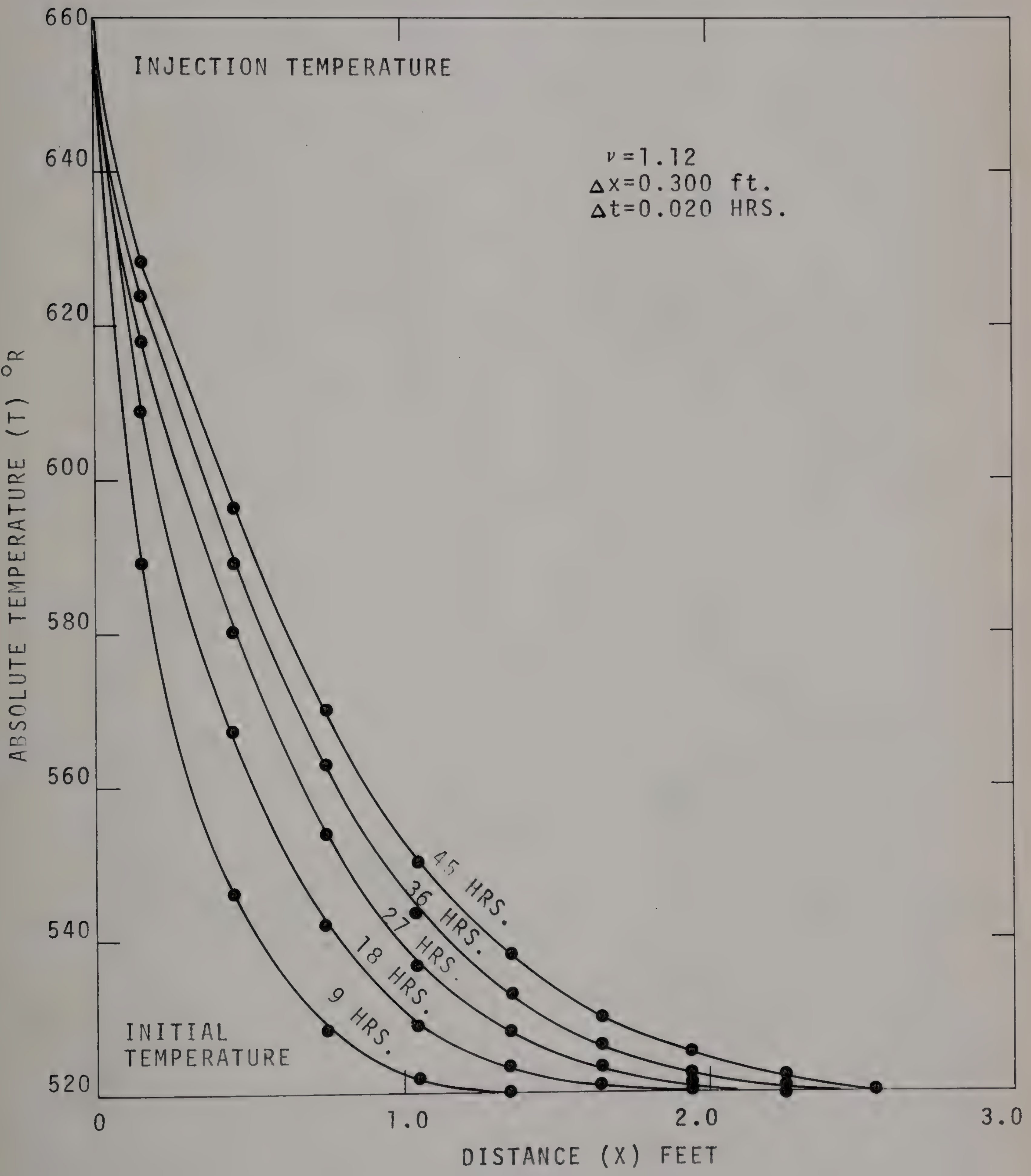
SATURATION DISTRIBUTIONS

FIGURE 14



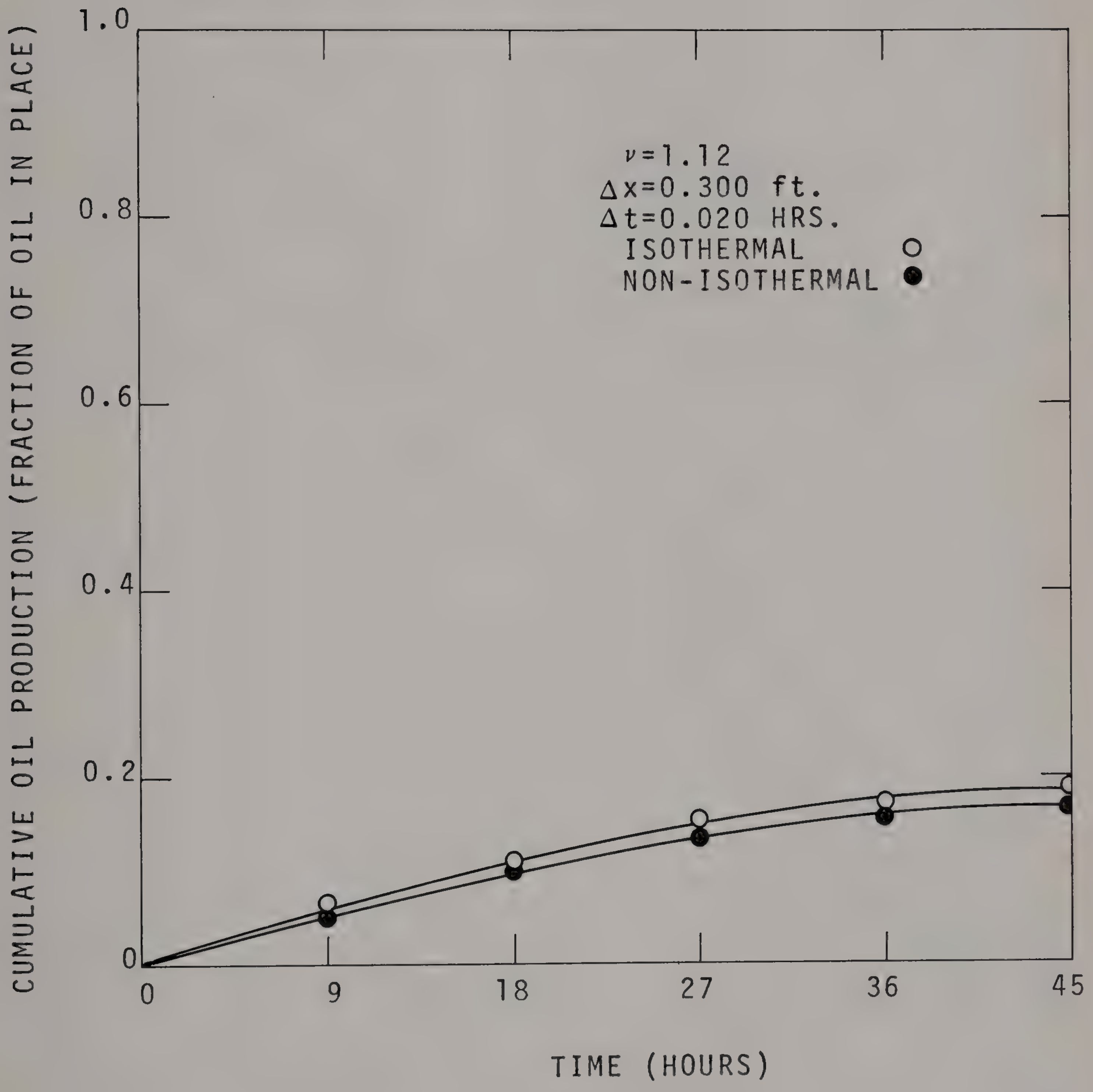
TEMPERATURE DISTRIBUTIONS

FIGURE 15



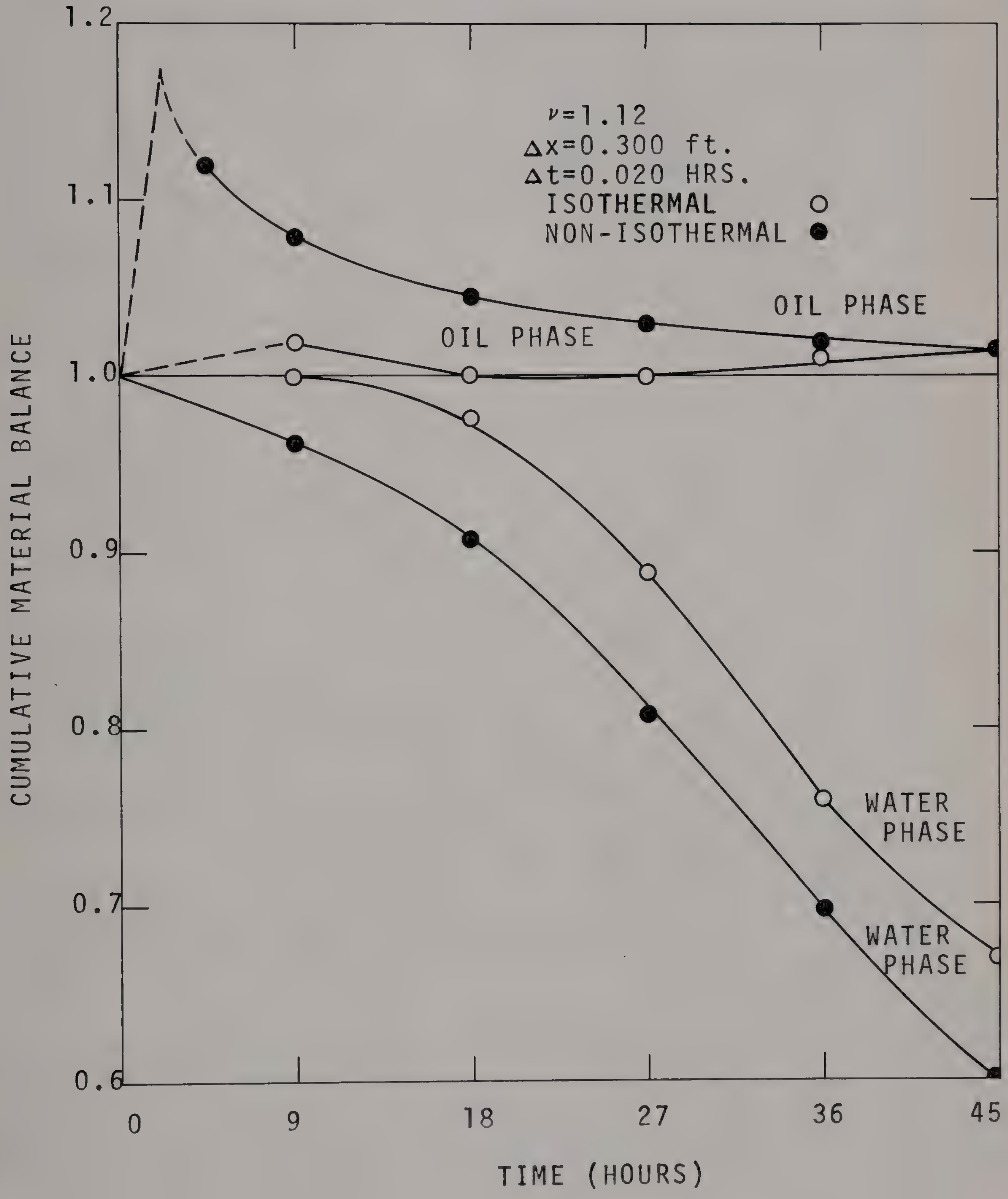
CUMULATIVE OIL PRODUCTION HISTORY

FIGURE 16



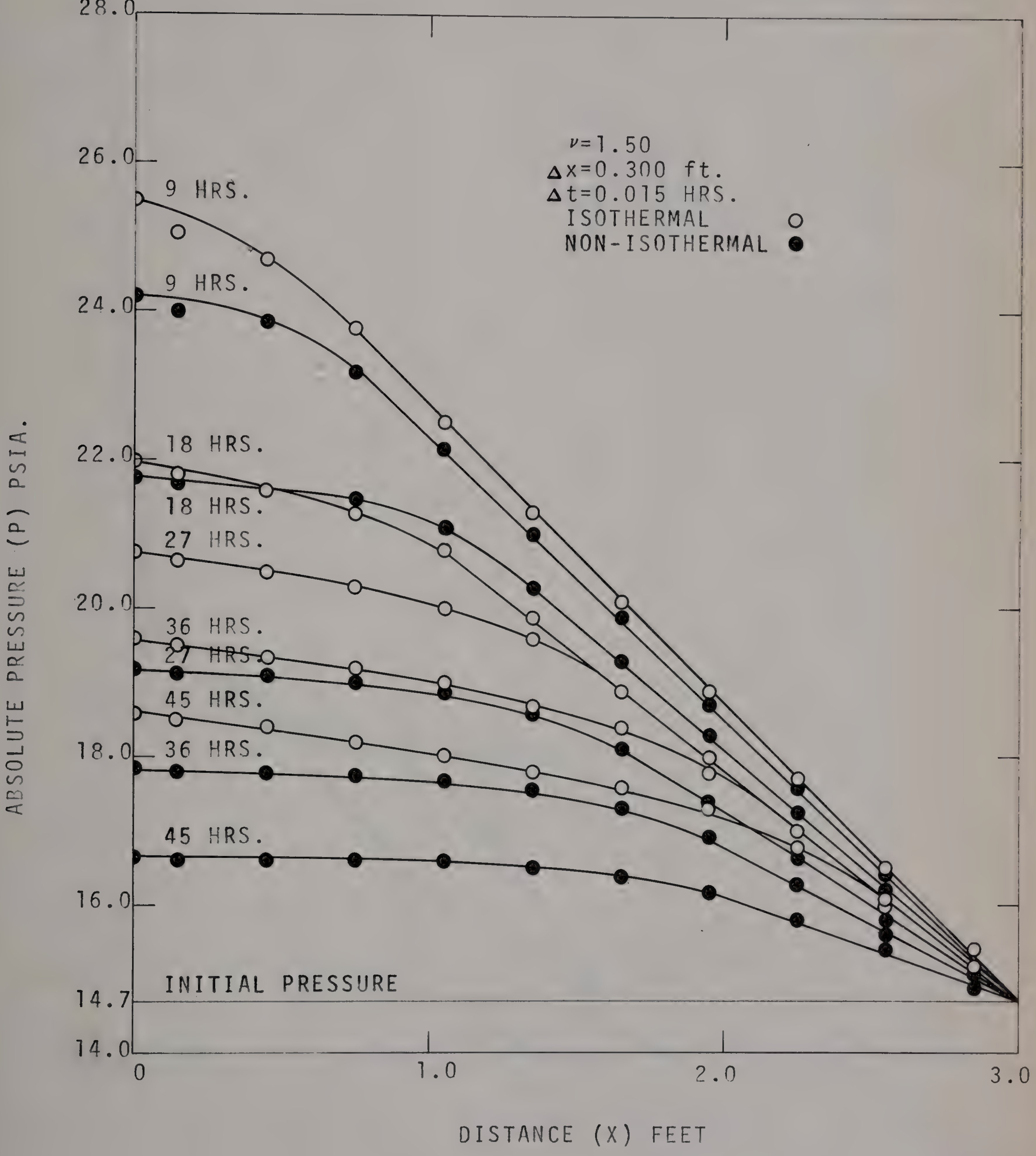
CUMULATIVE MATERIAL BALANCES

FIGURE 17



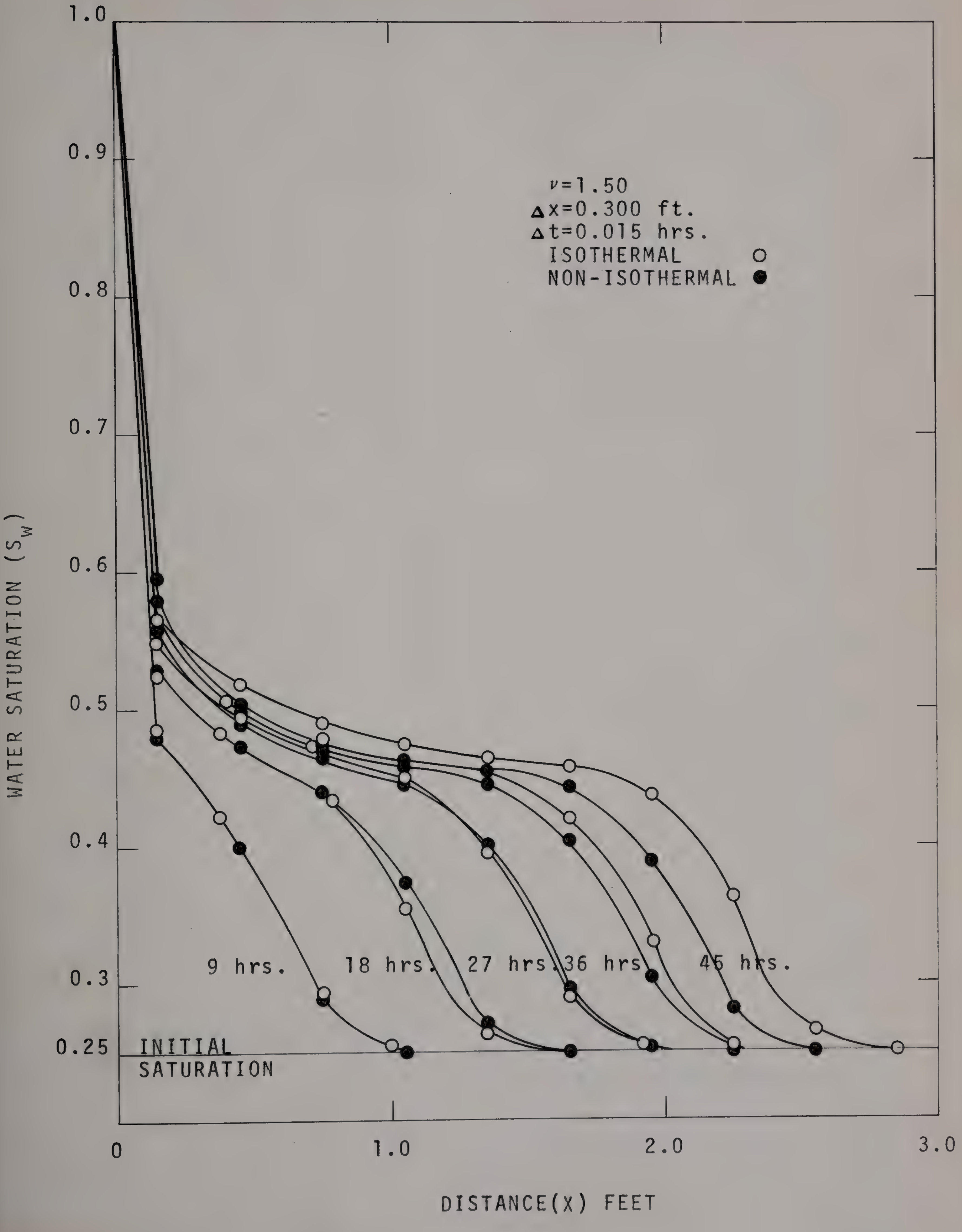
PRESSURE DISTRIBUTIONS

FIGURE 18



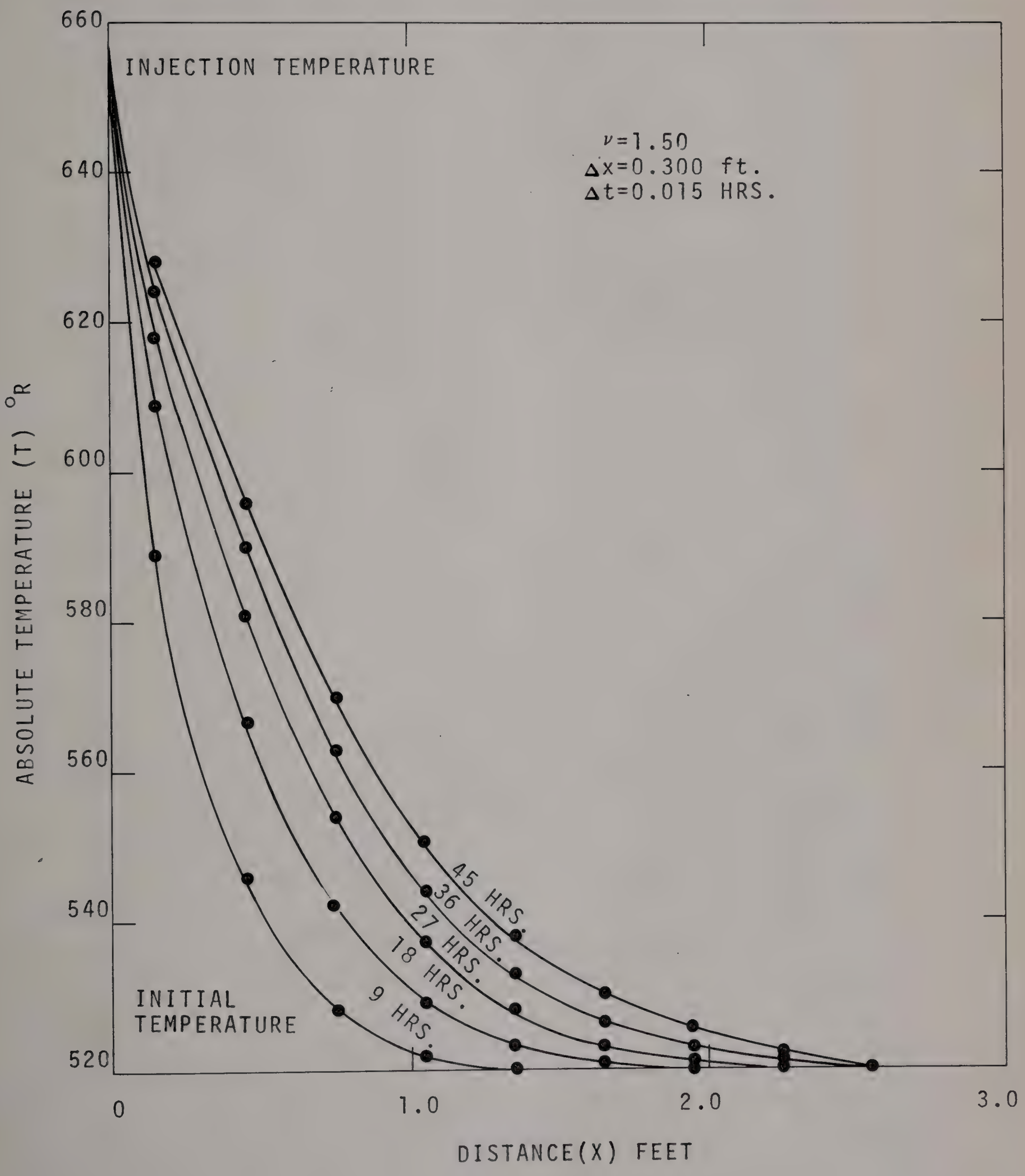
SATURATION DISTRIBUTIONS

FIGURE 19



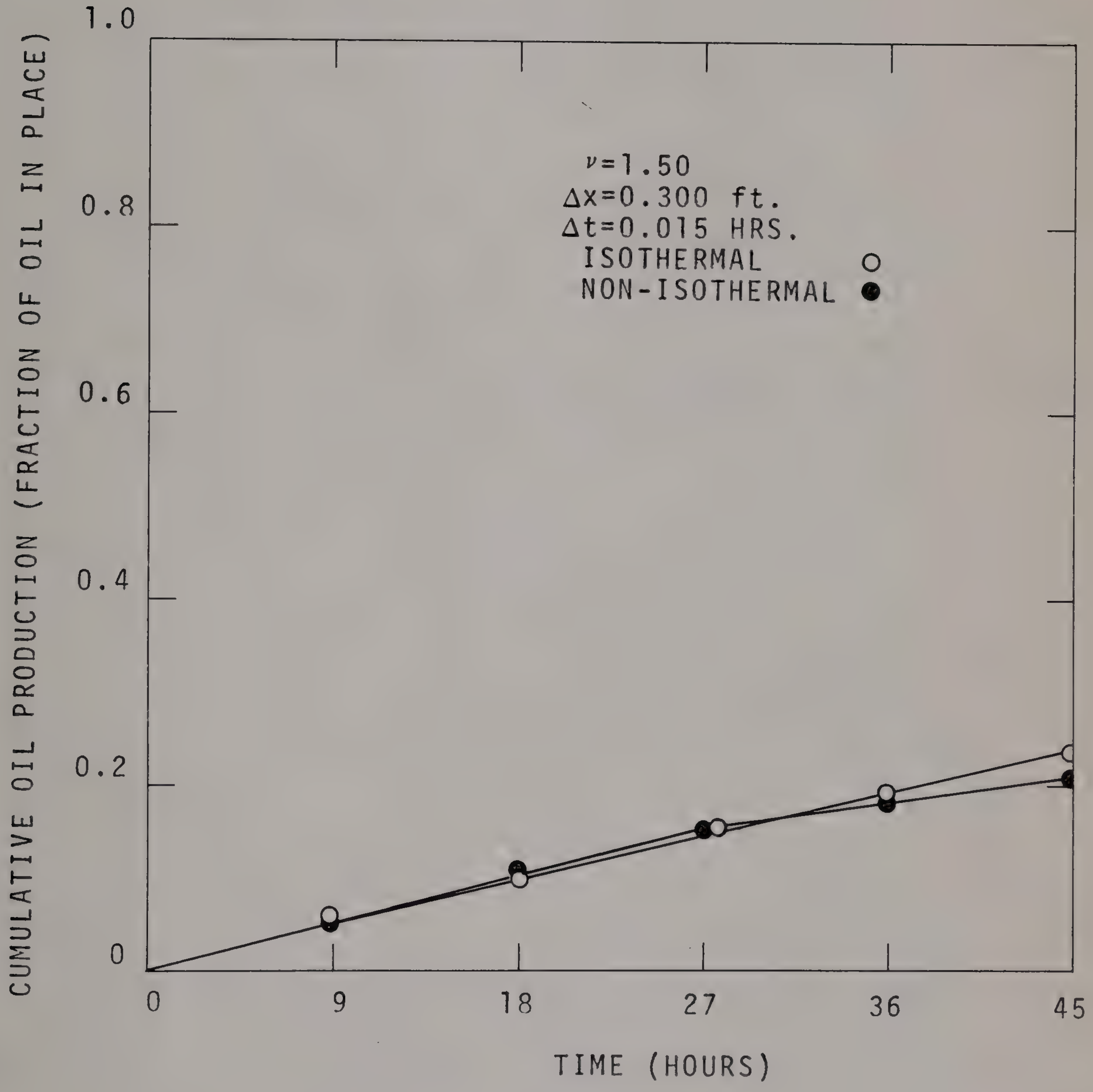
TEMPERATURE DISTRIBUTIONS

FIGURE 20



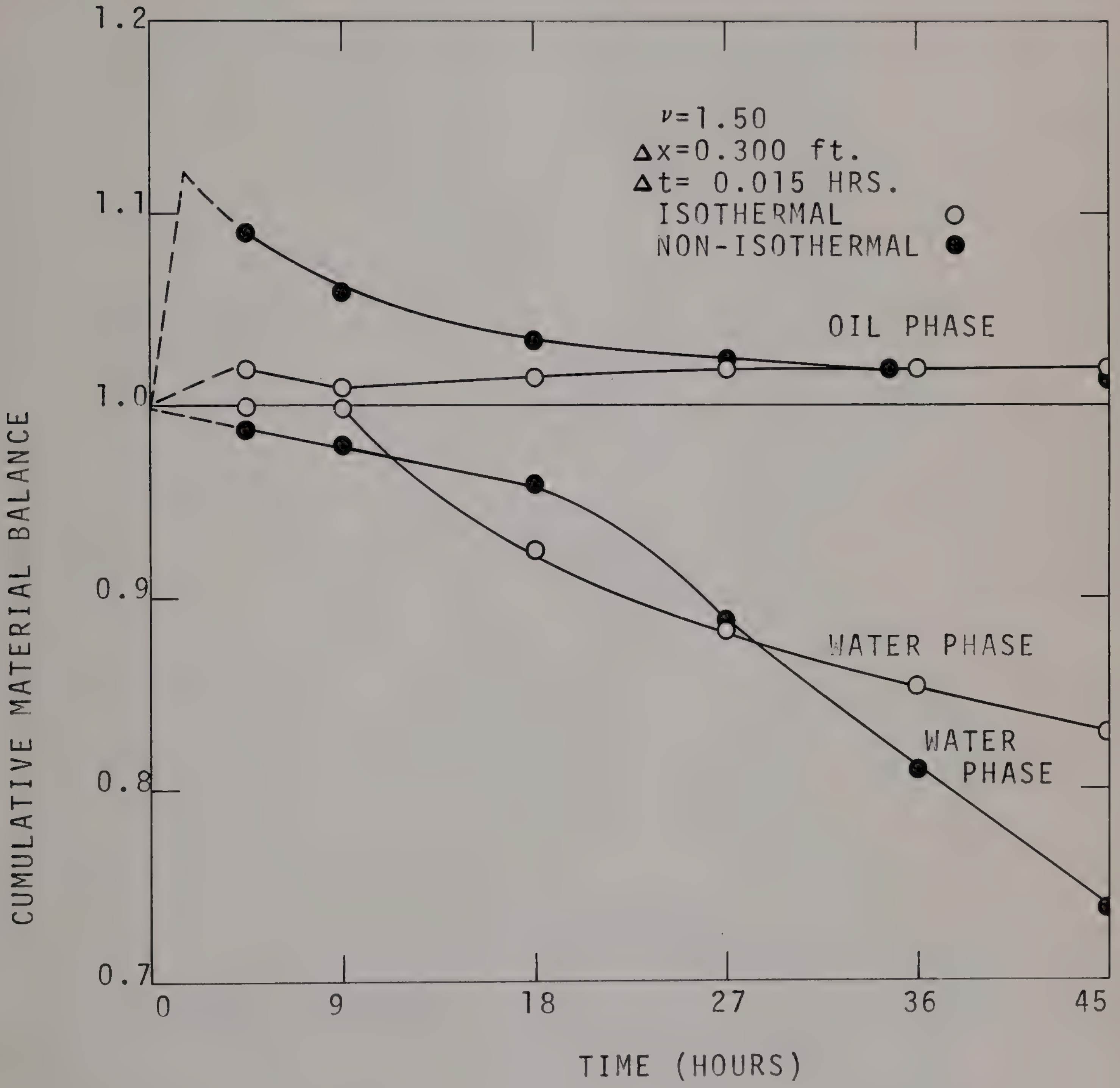
CUMULATIVE OIL PRODUCTION HISTORY

FIGURE 21



CUMULATIVE MATERIAL BALANCES

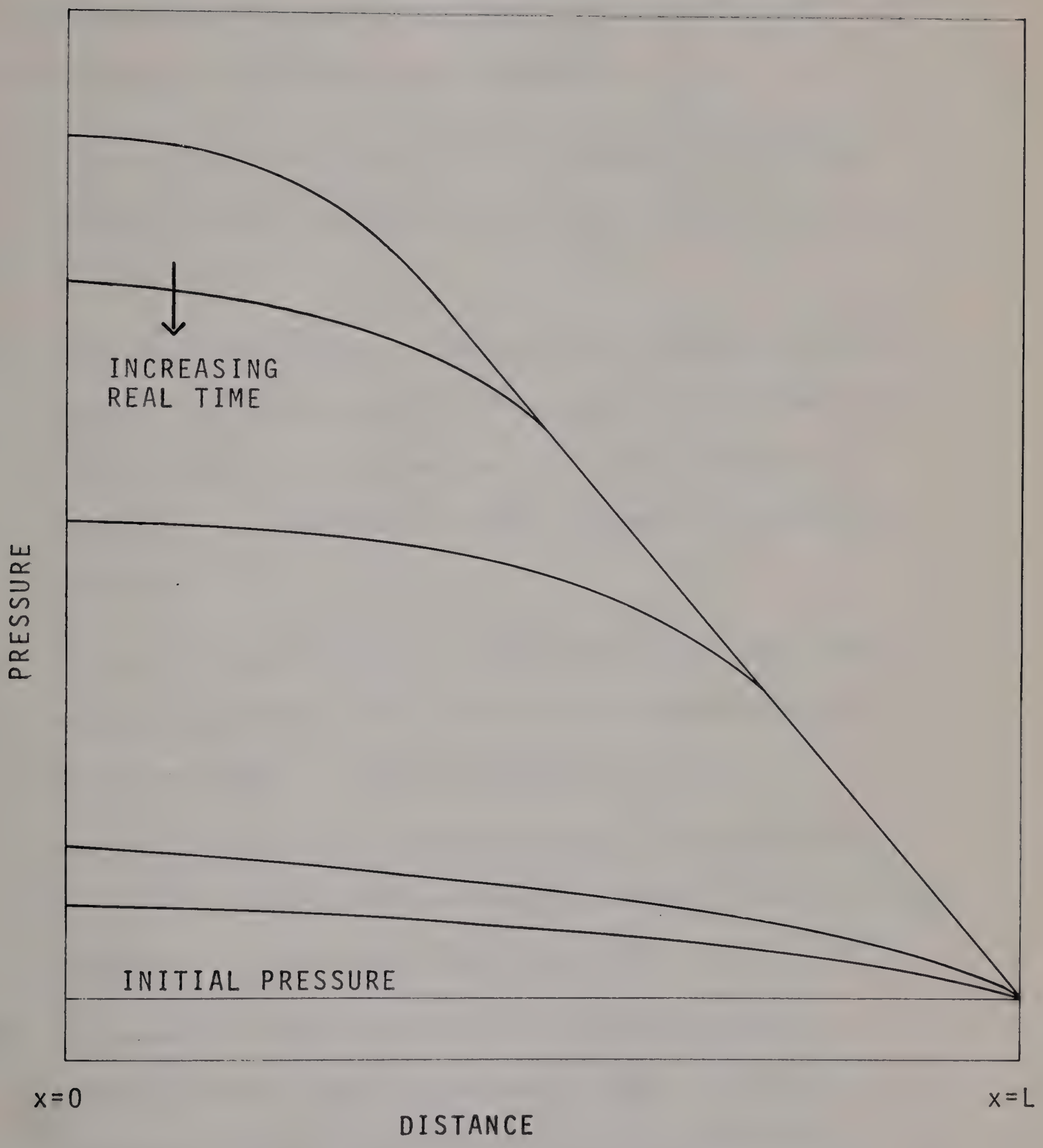
FIGURE 22



PRESSURE DISTRIBUTIONS

FIGURE 23

(SKETCH ONLY)
(REFERENCE 35)



CONCLUSIONS

Based on the work carried out in this study and the results obtained from specific example problems the following conclusions are stated:

(1) Mathematical modelling of a thermally stimulated reservoir model is possible by the method outlined in this work.

(2) Thermal stimulation of reservoirs containing highly temperature sensitive oil viscosities increases the oil recovery and allows use of lower injection pressures as compared to the isothermal injection problem.

(3) Although verification of the model and the results was not possible, the results are representative of those found in the literature to date.

(4) The alternating direction explicit procedure may be used to calculate a one-dimensional, compressible, isothermal, two phase flow problem.

(5) The greater the non-linearity of the fluid properties the more restrictive the choice of grid size spacing and time step size becomes.

The use of small grid size spacings and time step sizes will tend to eliminate convergence problems and the increased computing time required should not be restrictive with the new IBM-360-67 computing facilities.

RECOMMENDATIONS

(1) The model developed in this work could be extended to multiphase flow systems.

(2) The combined thermal and hydrodynamical aspect as well as a rigorous treatment of heat losses to the overburden and underburden could be considered simultaneously employing the same approach used to solve the simpler isothermal two phase flow problem in this work; that is, ADEP and the Newton-Raphson iterative technique.

(3) Due to the recent controversy about the use of ADEP it would be desirable to employ other numerical techniques to solve this problem and compare the results obtained by the various methods.

(4) In this work the same relative permeability correlations for k_{ro} and k_{rw} were used for the isothermal and nonisothermal systems. This may not be acceptable and one reason that actual recoveries may be higher for a thermal model is that the k_{ro} and k_{rw} data might be more favorable for the thermal system, and

hence work in this area might be considered.

(5) The mobility of the fluids (k_{ro}/μ_o and k_{rw}/μ_w) may be such that if the mobility of the water changes faster than the mobility of the oil during thermal stimulation, recoveries may actually be less for the nonisothermal system as compared to the isothermal system. A comparison could be possible by using this work for both favorable and unfavorable mobility ratios to see if the phenomena could actually be observed and predicted.

NOMENCLATURE

C = fluid compressibility, psia^{-1}

$CC1$ = field unit conversion constant = 2.637(10^{-4}) ft^2. cp./md.hr.

CO = cold oil zone

C_p = specific heat at constant pressure, BTU/lbm°R

C_v = specific heat at constant volume, BTU/lbm°R

CW = cold water zone

e = exponential = 2.171828

g = acceleration of gravity = 32.17 ft./sec^2.

G_w = mass injection rate of water, lbm./hr.ft^2.

G_{st} = mass injection rate of steam, lbm./hr.ft^2.

h = permeable sand thickness, ft.

HW = hot water zone

k = effective permeability, md.

K = absolute permeability

k_e = effective thermal conductivity, BTU/hr.ft.°R

L = length of model, ft.

P = absolute pressure, psia.

q = Darcy's Law velocity, ft./hr.

Q = volumetric flow rate, ft^3./hr.

q_t = heat flux, BTU/hr.ft^2.

r = r-direction coordinate, ft.

r_e = external reservoir radius, ft.

r_w = wellbore radius, ft.

S = fluid saturation, fraction

ST = steam zone

t = time, hrs.

T = temperature °F.

T_o = absolute temperature of the reservoir sand, °R

T_1 = absolute temperature of the overburden and underburden, °R

v = fluid velocity, ft./hr.

V = vaporization term

x = x-direction coordinate, ft.

y = y-direction coordinate, ft.

z = z-direction coordinate, ft.

GREEK LETTERS

α = thermal diffusivity = $k_e/\phi C_p$, ft^2/hr.

Δ = forward difference

θ = θ-direction coordinate

μ = fluid viscosity, cp.

π = 3.1416

ρ = density, lbm./ft^3.

τ = shearing stress

ϕ = effective porosity, fraction

SUBSCRIPTS

e = effective

i = initial

INJ = injection

j = jth grid intersection in the x-direction

o = oil

oi = initial oil conditions

OB = overburden

r = rock and relative

st = steam

w = water

wc = connate water

wi = initial water conditions

x = x-direction

y = y-direction

z = z-direction

SUPERSCRIPTS

(n) = (n)th time step n = 0, 1, 2, ...

(n+1) = (n+1)st time step n = 0, 1, 2, ...

(r) = (r)th recurrence parameter r = 0,1,2, ...

(r+1) = (r+1) recurrence parameter r = 0,1,2, ...

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APPENDIX A

DERIVATION OF THE STEAM DISPLACEMENT MATHEMATICAL MODEL

DERIVATION OF THE STEAM DISPLACEMENT MATHEMATICAL MODEL

Basis: Unit Reservoir Volume

The simultaneous flow of three fluids, oil, water, and steam may be described mathematically in terms of the continuity equation for each phase.

The continuity equation for a compressible fluid may be written as follows:

$$\begin{bmatrix}\text{Rate of change}\\ \text{of mass}\\ \text{per unit volume}\end{bmatrix} = \begin{bmatrix}\text{Net mass flux by}\\ \text{bulk transport}\\ \text{per unit volume}\end{bmatrix} + \begin{bmatrix}\text{Rate of mass}\\ \text{increase due to}\\ \text{vaporization per}\\ \text{unit volume}\end{bmatrix}$$

$$+ \begin{bmatrix}\text{Net mass flux by}\\ \text{molecular diffusion}\\ \text{per unit volume}\end{bmatrix}$$

or in mathematical notation, neglecting mass flux by molecular diffusion the three phase flow written for each phase separately is represented as:

Oil Phase

$$\nabla \cdot (\rho_o \vec{v}_o) + \phi \frac{\partial}{\partial t} (\rho_o S_o) = 0 \qquad \text{(A-1)}$$

Water Phase

$$\nabla \circ (\rho_w \vec{v}_w) - V_w(T_o, P) + \phi \frac{\partial}{\partial t} (\rho_w S_w) = 0 \qquad \text{(A-2)}$$

Steam Phase

$$\nabla \cdot (\rho_{st} \vec{v}_{st}) + V_w (T_o, P) + \phi \frac{\partial}{\partial t} (\rho_{st} S_{st}) = 0 \qquad \text{(A-3)}$$

T_o (refers to reservoir temperature)

The velocity of each phase may be represented by Darcy's Law neglecting gravity effects (horizontal flow) and capillary forces.

Oil Phase

$$\vec{q}_o = \frac{-k_o}{\mu_o} \frac{\partial P}{\partial r} = \frac{\vec{Q}_o}{2\pi rh} = \vec{v}_o \qquad \text{(A-4)}$$

Water Phase

$$\vec{q}_w = \frac{-k_w}{\mu_w} \frac{\partial P}{\partial r} = \frac{\vec{Q}_w}{2\pi rh} = \vec{v}_w \qquad \text{(A-5)}$$

Steam Phase

$$\vec{q}_{st} = \frac{-k_{st}}{\mu_{st}} \frac{\partial P}{\partial r} = \frac{\vec{Q}_{st}}{2\pi rh} = \vec{v}_{st} \qquad \text{(A-6)}$$

Since a radial system is to be considered, the divergence $(\nabla \circ \vec{A})$ in cylindrical coordinates may be written as,

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (rA_r) + \frac{1}{r} \frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \qquad \text{(A-7)}$$

Combining equations (A-4), (A-5), and (A-6) with equations (A-1), (A-2), and (A-3), and applying equation (A-7) for horizontal, one-dimensional flow the hydrodynamical aspects of the model are developed for each phase.

Oil Phase

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r\rho_o k_o}{\mu_o} \frac{\partial P}{\partial r} \right) - \phi \frac{\partial}{\partial t} (\rho_o S_o) = 0 \qquad \text{(A-8)}$$

Water Phase

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r\rho_w k_w}{\mu_w} \frac{\partial P}{\partial r} \right) - V_w(T_o, P) - \phi \frac{\partial}{\partial t} (\rho_w S_w) = 0 \qquad \text{(A-9)}$$

Steam Phase

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r\rho_{st} k_{st}}{\mu_{st}} \frac{\partial P}{\partial r} \right) + V_w(T_o, P) - \phi \frac{\partial}{\partial t} (\rho_{st} S_{st}) = 0 \qquad \text{(A-10)}$$

From material balance considerations,

$$S_o + S_w + S_{st} = 1.0 \qquad \text{(A-11)}$$

The Thermal Energy Equation

The thermal energy equation as stated by Bird, Stewart, and Lightfoot (1) is;

$$\begin{bmatrix}\text{Rate of gain of}\\ \text{internal energy}\\ \text{per unit volume}\\ \end{bmatrix} = \begin{bmatrix}\text{Rate of internal}\\ \text{energy input by}\\ \text{conduction per}\\ \text{unit volume}\end{bmatrix} + \begin{bmatrix}\text{Reversible rate of}\\ \text{internal energy in-}\\ \text{crease per unit}\\ \text{volume by compression}\end{bmatrix}$$

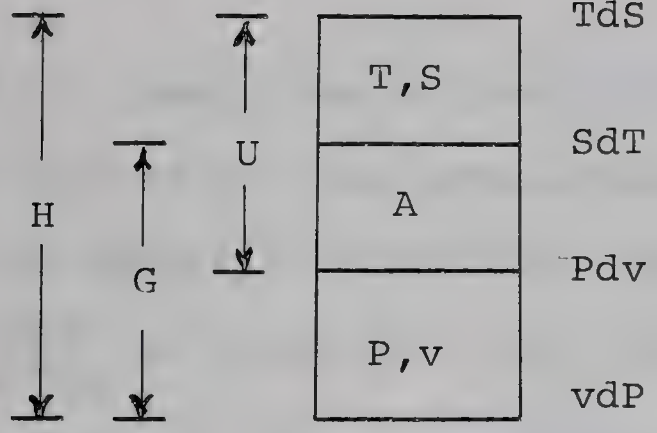
$$+ \begin{bmatrix}\text{irreversible rate}\\ \text{of internal energy}\\ \text{increase per unit}\\ \text{volume by viscous}\\ \text{dissipation}\end{bmatrix} + \begin{bmatrix}\text{net gain due to}\\ \text{condensation}\\ \text{per unit volume}\\ \\ \end{bmatrix}$$

In mathematical terms the equation is written:

$$\rho \frac{DU}{Dt} = - \nabla \cdot \vec{q}_t - P(\nabla \cdot \vec{v}) - \vec{\tau} : \nabla \vec{v} + V(T_o, P) \qquad \text{(A-12)}$$

The internal energy (U) is not a convenient property for practical engineering problems. Manipulation of the basic thermodynamic functions enables the transformation of equation (A-12) to an equation involving the system temperature and heat capacity rather than internal energy.

THERMODYNAMIC RELATIONS



$$dU = T\,dS - Pdv \qquad \text{(A-13)}$$

$$dH = T\,dS + vdP \qquad \text{(A-14)}$$

$$dA = -S\,dT - Pdv \qquad \text{(A-15)}$$

$$dG = -S\,dT + vdP \qquad \text{(A-16)}$$

U = internal energy

H = enthalpy

A = total work Helmholtz Function

G = Gibb's Free Energy

S = entropy

T,P,v = absolute temperature, absolute pressure and specific volume

Assume $S = f(v,T)$

$$\text{then } dS = \left(\frac{\partial S}{\partial v}\right)_T dv + \left(\frac{\partial S}{\partial T}\right)_v dT$$

By definition, $\left(\frac{\partial S}{\partial T}\right)_v = \frac{C_v}{T}$ where C_v is the specific heat at constant volume.

therefore $$dS = \left(\frac{\partial S}{\partial v}\right)_T dv + \frac{C_v}{T} dT \qquad \text{(A-17)}$$

It is desirable to have equation (A-17) in terms of pressure (P) and temperature (T) rather than entropy (S) and specific volume (v). An expression for the derivative $\left(\frac{\partial S}{\partial v}\right)_T$ in terms of P and T may be obtained from a Maxwell Relation. Since equations (A-13) through (A-16) involve point (intensive) properties (independent of path) they are exact differentials and therefore of the general form,

$$dz = Mdx + Ndy$$

such that $\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$

Proof: Green's Theorem (continuous functions)

$$z = f(x,y)$$

$$dz = \left(\frac{\partial z}{\partial y}\right)_x dy + \left(\frac{\partial z}{\partial x}\right)_y dx$$

Let $M = \frac{\partial z}{\partial x}$ and $N = \frac{\partial z}{\partial y}$

Then $\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial z}{\partial x \partial y}$ and $\left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial z}{\partial y \partial x}$

Applying Green's Theorem to equation (A-15), the desired Maxwell Relation is obtained.

$$\left(\frac{\partial S}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v \qquad (A\text{-}18)$$

Substitute equation (A-18) into equation (A-17).

$$dS = \left(\frac{\partial P}{\partial T}\right)_v dv + \frac{C_v}{T} dT \qquad (A\text{-}19)$$

Substitution of equation (A-19) into equation (A-13) yields an expression for the internal energy as a function of pressure, temperature, and heat capacity.

$$dU = \left[-P + T_o \left(\frac{\partial P}{\partial T}\right)_v \right] dv + C_v \, dT_o$$

The substantial derivative of U, multiplied by the density is:

$$\rho \frac{DU}{Dt} = \left[-P + T_o \left(\frac{\partial P}{\partial T}\right)_v \right] \rho \frac{Dv}{Dt} + \rho\, C_v \frac{DT_o}{Dt} \qquad \text{(A-20)}$$

The specific volume (v) may be written as

$$v = \frac{1}{\rho}$$

then $\rho \frac{Dv}{Dt} = \rho \frac{D}{Dt}\left(\frac{1}{\rho}\right) = \rho \left(\frac{-1}{\rho^2}\right) \frac{D\rho}{Dt} = \frac{-1}{\rho} \frac{D\rho}{Dt}$

But the continuity equation states,

$$\frac{-1}{\rho} \frac{D\rho}{Dt} = (\nabla \circ \vec{v}) \qquad \text{(A-21)}$$

Substitute equation (A-21) into equation (A-20) to obtain:

$$\rho \frac{DU}{Dt} = \left[-P + T_o \left(\frac{\partial P}{\partial T}\right)_v \right] \nabla \circ \vec{v}_{st} + \rho\, C_v \frac{DT_o}{Dt} \qquad \text{(A-22)}$$

Substitution of equation (A-22) into equation (A-12) renders the final form of the thermal energy equation,

$$\rho\, C_v \frac{DT_o}{Dt} = - \nabla \cdot \vec{q}_t - T_o \left(\frac{\partial P}{\partial T}\right)_v \nabla \circ \vec{v}_{st} - \vec{\tau} : \nabla \vec{v} + V(T_o, P) \qquad \text{(A-23)}$$

where $$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z \frac{\partial}{\partial z} \qquad (v = \text{velocity})$$

$$\rho C_v = (S_o \rho_o C_{v_o} + S_w \rho_w C_{v_w} + S_{st} \rho_{st} C_{v_{st}})\phi + \rho_r C_{p_r}(1-\phi)$$

$$-\nabla \cdot \vec{q}_t = \nabla \cdot (k_e \nabla T) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\, k_e \frac{\partial T}{\partial r} \right) \qquad (\text{one-dimension})$$

$$\nabla \cdot \vec{v}_{st} = \frac{1}{r}\frac{\partial}{\partial r}(r\, \vec{v}_{st})$$

The mechanical energy equation as taken from Bird, Stewart and Lightfoot $^{(1)}$ is,

$$\rho \frac{D}{Dt}\left(\frac{v^2}{2} \right) = P(\nabla \cdot \vec{v}) - \nabla \circ (Pv) + \rho(\vec{v} \circ \vec{g}) - (\nabla \circ [\tau \cdot \vec{v}]) + (\vec{\tau} : \nabla \vec{v}) \qquad (A-24)$$

A comparison of the mechanical energy equation (A-24) with the thermal energy equation (A-12) reveals the terms $P(\nabla \cdot \vec{v})$ and $(\tau : \nabla \vec{v})$ are common to both equations and appear with the opposite signs in the two equations. Therefore these terms describe the interconversion of mechanical and thermal energy. The term $P(\nabla \circ \vec{v})$ can be positive or negative, depending upon whether the fluid is expanding or contracting; hence it represents a reversible mode of

interchange. The term $(-\vec{\tau}:\nabla\vec{v})$ is always positive and therefore represents an irreversible degradation of mechanical to thermal energy. These viscous dissipation terms may usually be neglected except for systems with large velocity gradients (1).

Heat Losses to Overburden and Underburden

To complete the mathemcatical model the heat losses to the overburden and underburden may be specified by Fourier's Law written in one-dimension.

$$\frac{\partial^2 T_1}{\partial z^2} = \frac{1}{\alpha_{OB}} \frac{\partial T_1}{\partial t} \qquad \text{(A-25)}$$

where T_1 refers to overburden or underburden temperatures.

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(1) "Transport Phenomena", Bird, R.B., Steward, W.E., and Lightfoot, E.N., John Wiley and Sons, Inc., Sixth Printing, 1965.

APPENDIX B

DERIVATION OF THE HOT-WATER INJECTION MODEL

DERIVATION OF THE HOT-WATER INJECTION MODEL

Basis: unit reservoir volume

For a linear hot-water injection process the combined continuity-Darcy flow equations (A-8) and (A-9) assume the form:

Oil Phase:

$$\frac{\partial}{\partial x}\left(\frac{\rho_o k_o}{\mu_o}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\frac{\partial}{\partial t}(\rho_o S_o) \tag{B-1}$$

Water Phase

$$\frac{\partial}{\partial x}\left(\frac{\rho_w k_w}{\mu_w}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\frac{\partial}{\partial t}(\rho_w S_w) \tag{B-2}$$

Equation (A-11) reduces to

$$S_o + S_w = 1.0 \tag{B-3}$$

The densities of oil and water as functions of pressure may be written as:

$$\rho_o = \rho_o^* \, e^{C_o (P-P^*)} \tag{B-4}$$

$$\rho_w = \rho_w^* \, e^{C_w (P-P^*)} \tag{B-5}$$

*refers to a reference value.

Mathematical Manipulations

$$\begin{aligned}\frac{\partial}{\partial x}\left(\rho_o \frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) &= \rho_o \frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) + \left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right)\frac{\partial \rho_o}{\partial x} \\ &= \rho_o \frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) + \left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right)\frac{\partial \rho_o}{\partial P}\frac{\partial P}{\partial x} \\ &= \rho_o \frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) + \frac{k_o}{\mu_o}\left(\frac{\partial P}{\partial x}\right)^2 \frac{\partial \rho_o}{\partial P} \qquad \text{(B-6)}\end{aligned}$$

Differentiate equation (B-4) with respect to pressure,

$$\frac{\partial \rho_o}{\partial P} = \frac{\partial}{\partial P}\left[\rho_o{}^* \; e^{C_o(P-P^*)}\right] = C_o \, \rho_o$$

and substitute into equation (B-6)

$$\frac{\partial}{\partial x}\left(\frac{\rho_o k_o}{\mu_o}\frac{\partial P}{\partial x}\right) = \rho_o \frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) + \frac{k_o}{\mu_o}\,(C_o\rho_o)\left(\frac{\partial P}{\partial x}\right)^2 \qquad \text{(B-7)}$$

Since $C_o << 1.0$ or $C_oP <<< 1.0$ (Reference **)

** Dranchuk, P.M. and Quon, D.: "Analaysis of the Darcy-Continuity Equation", Petrol. Soc. CIM, 17th Annual Meeting, Edmonton, May (1966)

$$\frac{k_o}{\mu_o}\;(C_o\rho_o)\left(\frac{\partial P}{\partial x}\right)^2 \;<<\; \rho_o\,\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\,\frac{\partial P}{\partial x}\right)$$

Equation (B-7) therefore reduces to

$$\frac{\partial}{\partial x}\left(\frac{\rho_o k_o}{\mu_o}\,\frac{\partial P}{\partial x}\right) = \rho_o\,\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\,\frac{\partial P}{\partial x}\right) \tag{B-8}$$

Also, $\frac{\partial}{\partial t}(\rho_o S_o) = \rho_o\,\frac{\partial S_o}{\partial t} + S_o\,\frac{\partial \rho_o}{\partial t}$ (B-9)

Differentiate equation (B-4) with respect to time,

$$\frac{\partial \rho_o}{\partial t} = \frac{\partial}{\partial t}\left[\rho_o^{*}\;e^{C_o(P-P^{*})}\right] = C_o\rho_o\,\frac{\partial P}{\partial t}$$

and substitute into equation (B-9)

$$\frac{\partial}{\partial t}(\rho_o S_o) = \rho_o\,\frac{\partial S_o}{\partial t} + \rho_o C_o S_o\,\frac{\partial P}{\partial t} \tag{B-10}$$

Substitute equations (B-8) and (B-10) into equation (B-1).

$$\rho_o\,\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\,\frac{\partial P}{\partial x}\right) = \frac{\phi\rho_o}{CCl}\left[\frac{\partial S_o}{\partial t} + C_o\,S_o\,\frac{\partial P}{\partial t}\right]$$

Since $\rho_o \neq 0$, divide through by ρ_o.

$$\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\left[\frac{\partial S_o}{\partial t} + C_o S_o \frac{\partial P}{\partial t}\right] \qquad \text{(B-11)}$$

Since the function definitions for the density of oil and water are identical the above analysis for the oil phase is valid for the water phase and the transformation of equation (B-2) may be written directly.

$$\frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\left[\frac{\partial S_w}{\partial t} + C_w S_w \frac{\partial P}{\partial t}\right] \qquad \text{(B-12)}$$

Applying equation (B-3) and noting $\frac{\partial}{\partial t}(1-S_o) = \frac{\partial}{\partial t}(-S_o)$ equation (B-12) becomes,

$$\frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\left[\frac{\partial}{\partial t}(-S_o) + C_w(1-S_o)\frac{\partial P}{\partial t}\right] \qquad \text{(B-13)}$$

The Thermal Energy Equation

The thermal energy equation (A-23) neglecting viscous dissipation terms and the reversible interchange of

mechanical and thermal energy due to compression or expansion becomes:

$$\rho C_v \frac{DT}{Dt} = -\nabla \cdot \vec{q}_t \tag{B-14}$$

In cartesian coordinates the substantial derivative is:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \tag{B-15}$$

Applying equation (B-15) to equation (B-14) for one-dimensional, horizontal flow yields,

$$\rho C_v \frac{\partial T}{\partial t} = - \rho C_v v_x \frac{\partial T}{\partial x} - \nabla \cdot \vec{q}_t \tag{B-16}$$

The velocity v_x is given by Darcy's Law based on the effective flow area

$$v_x = - \frac{CC1}{\phi S_x} \frac{k_x}{\mu_x} \frac{\partial P}{\partial x}$$

and Fourier's law may be used to represent the heat flux $\vec{q}_t$

$$\vec{q}_t = - k_e \frac{\partial T}{\partial x}$$

and is based on the effective flow area. ($A_{eff} = A\phi S_x$)

Assume C_v (liquid or solid) $\simeq$ C_p (liquid or solid)

The energy equation may now be written for the separate phases.

Oil Phase

$$\phi S_o \rho_o C_{p_o} \frac{\partial T_o}{\partial t} = (CC1)\ \rho_o C_{p_o} \frac{k_o}{\mu_o} \frac{\partial P}{\partial x} \frac{\partial T_o}{\partial x} + \frac{\partial}{\partial x} \left(k_{e_o} \frac{\partial T_o}{\partial x} \right) \quad \text{(B-17)}$$

Water Phase

$$\phi (1-S_o) \rho_w C_{p_w} \frac{\partial T_w}{\partial t} = (CC1) \rho_w C_{p_w} \frac{k_w}{\mu_w} \frac{\partial P}{\partial x} \frac{\partial T_w}{\partial x} + \frac{\partial}{\partial x} \left(k_{e_w} \frac{\partial T_w}{\partial x} \right) \quad \text{(B-18)}$$

Solid Phase

$$(1-\phi)\ \rho_r C_{p_r} \frac{\partial T_r}{\partial t} = \frac{\partial}{\partial x} \left(k_{e_r} \frac{\partial T_r}{\partial x} \right) \quad \text{(B-19)}$$

Combining equations (B-17), (B-18), and (B-19) and assuming $T_o = T_w = T_r = T$ (see Appendix D) and

$$\rho C_p = [\rho_w C_{p_w} (1-S_o) + \rho_o C_{p_o} S_o]\ \phi + \rho_r C_{p_r} (1-\phi) \quad \text{(B-20)}$$

results in the final form of the thermal energy equation.

$$\rho C_p \frac{\partial T}{\partial t} = CC1 \left\{ \left[\rho_o C_{p_o} \frac{k_o}{\mu_o} + \rho_w C_{p_w} \frac{k_w}{\mu_w} \right] \frac{\partial P}{\partial x} \right\} \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(k_e \frac{\partial T}{\partial x} \right) \quad \text{(B-21)}$$

The heat losses to the overburden and underburden are neglected in this model.

APPENDIX C

DISCRETIZATION OF PARTIAL DIFFERENTIAL EQUATIONS AND METHODS OF SOLUTION

DISCRETIZATION OF PARTIAL DIFFERENTIAL EQUATIONS AND METHODS OF SOLUTION

ALTERNATING DIRECTION EXPLICIT PROCEDURE (ADEP) -- ONE-DIMENSION

From Appendix B, the continuity-flow equations (B-11) and (B-13) are:

Oil

$$\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\left[\frac{\partial S_o}{\partial t} + C_oS_o\frac{\partial P}{\partial t}\right] \qquad (C-1)$$

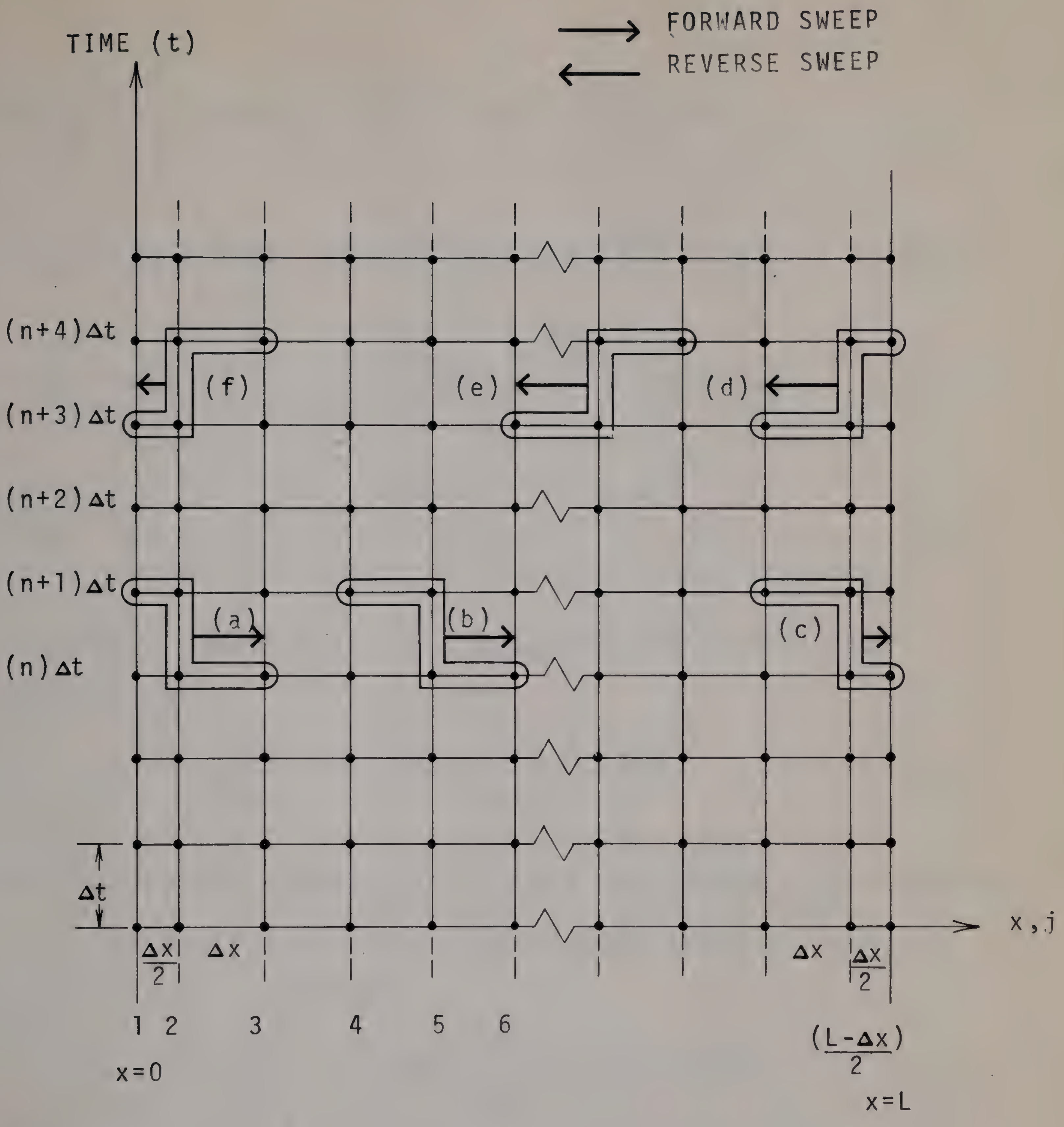
Water

$$\frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CC1}\left[\frac{\partial}{\partial t}(-S_o) + C_w(1-S_o)\frac{\partial P}{\partial t}\right] \qquad (C-2)$$

Approximations to the spatial derivatives may be obtained from the use of Taylor's Series.

$$f(x+\Delta x) = f(x) + \Delta x\, f'(x) + \frac{(\Delta x)^2}{2!} f''(x) + \frac{(\Delta x)^3}{3!} f'''(x) + \ldots$$

Consider any point j on a grid spacing as shown in Figure (C-1). The following expansions may be written to third order correctness.



ONE-DIMENSIONAL ALTERNATING DIRECTION EXPLICIT PROCEDURE -- GRID POINTS AND TIME LEVELS FOR THE FORWARD AND REVERSE SWEEP.

FIGURE C-1

$$T_{j+1} = T_j + \Delta x \; T'_j + \frac{(\Delta x)^2}{2} \; T''_j + \frac{(\Delta x)^3}{6} \; T'''_j \qquad \text{(C-3)}$$

$$T_{j+2} = T_j + 2(\Delta x) \; T'_j + \frac{(2\Delta x)^2}{2} \; T''_j + \frac{(2\Delta x)^3}{6} \; T'''_j \qquad \text{(C-4)}$$

$$T_{j-1} = T_j - (\Delta x) \; T'_j + \frac{(\Delta x)^2}{2} \; T''_j - \frac{(\Delta x)^3}{6} \; T'''_j \qquad \text{(C-5)}$$

$$T_{j-2} = T_j - 2(\Delta x) \; T'_j + \frac{(2\Delta x)^2}{2} \; T''_j - \frac{(2\Delta x)^3}{6} \; T'''_j \qquad \text{(C-6)}$$

By adding equation (C-3) to equation (C-5) it can be shown that

$$T''_j = \left(\frac{\partial^2 T}{\partial x^2}\right)_j = \frac{T_{j+1} - 2T_j + T_{j-1}}{(\Delta x)^2} \qquad \text{(C-7)}$$

By multiplying equation (C-5) by 8 and adding it to equation (C-4) an expression for the first derivative of T with respect to x is obtained.

$$T'_j = \left(\frac{\partial T}{\partial x}\right)_j = \frac{-T_{j+2} + 6T_{j+1} - 3T_j - 2T_{j-1}}{6(\Delta x)} \qquad \text{(C-8)}$$

Multiplication of equation (C-5) by 8 and subtraction of equation (C-6) yields an expression for the first derivative

of T with respect to x incorporating two backward points and one forward point.

$$T'_j = \left(\frac{\partial T}{\partial x}\right)_j = \frac{2T_{j+1}+3T_j-6T_{j-1}+T_{j-2}}{6(\Delta x)} \qquad \text{(C-9)}$$

Equations (C-7) to (C-9) are the approximations to the spatial derivatives of pressure and temperature to be used in this study.

Discretization of the hydrodynamic equations (C-1) and (C-2) using equation (C-7) yields equations (C-10) and (C-11).

Oil

$$\left[\left(\frac{k_o}{\mu_o}\right)_{j-1/2} (P_{j-1}-P_j)\right] + \left[\left(\frac{k_o}{\mu_o}\right)_{j+1/2} (P_{j+1}-P_j)\right] = \frac{\phi(\Delta x)^2}{CC1}\left[\frac{\partial S_o}{\partial t} + C_o S_o \frac{\partial P}{\partial t}\right] \qquad \text{(C-10)}$$

Water

$$\left[\left(\frac{k_w}{\mu_w}\right)_{j-1/2} (P_{j-1}-P_j)\right] + \left[\left(\frac{k_w}{\mu_w}\right)_{j+1/2} (P_{j+1}-P_j)\right] = \frac{\phi(\Delta x)^2}{CC1}\left[\frac{\partial}{\partial t}(-S_o)+C_w(1-S_o)\frac{\partial P}{\partial t}\right]$$

(C-11)

At this point, the particular method used will dictate the time levels at which the various terms must be calculated. The alternating direction explicit procedure (ADEP) in one-dimension as shown schematically in Figure (C-1) was applied to equations (C-10) and (C-11). The boundaries at x = 0 and x = L are specified and it is desired to obtain a pressure, saturation and temperature distribution at all grid points.

GRID POINT (j=2) - FORWARD SWEEP - FIGURE (C-1-a)

Oil

$$\frac{\left(\frac{k_o}{\mu_o}\right)^{(n)}_{j+1/2} \left.\frac{\partial P}{\partial x}\right|^{(n)}_{j+1/2} - \left(\frac{k_o}{\mu_o}\right)^{(n+1)}_{j-1} \left.\frac{\partial P}{\partial x}\right|^{(n+1)}_{j-1}}{\Delta x}$$

$$= \frac{\phi}{CC1(\Delta t)}\left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)}\left(P_j^{(n+1)} - P_j^{(n)}\right)\right] \quad \text{(C-12)}$$

$$-\left(\frac{k_o}{\mu_o}\right)_{j-1} \left.\frac{\partial P}{\partial x}\right|_{j-1} = \left.\frac{G_o}{CC1\ \rho_o}\right|_{x=0} = 0.0 \text{ (Boundary Condition)} \quad \text{(C-13)}$$

$$\left(\frac{k_o}{\mu_o}\right)^{(n)}_{j+1/2} \left(P_{j+1} - P_j\right)^{(n)} = \frac{\phi(\Delta x)^2}{CC1\ \Delta t}\left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)}\left(P_j^{(n+1)} - P_j^{(n)}\right)\right] \quad \text{(C-14)}$$

Water

$$\frac{\left(\frac{k_w}{\mu_w}\right)_{j+1/2}^{(n)} \left.\frac{\partial P}{\partial x}\right|_{j+1/2}^{(n)} - \left(\frac{k_w}{\mu_w}\right)_{j-1}^{(n+1)} \left.\frac{\partial P}{\partial x}\right|_{j-1}^{(n+1)}}{\Delta x}$$

$$= \frac{\phi}{CC1(\Delta t)} \left[S_{o_j}^{(n)} - S_{o_j}^{(n+1)} + C_w(1-S_{o_j}^{(n+1/2)})(P_j^{(n+1)} - P_j^{(n)}) \right] \qquad (C-15)$$

$$-\left(\frac{k_w}{\mu_w}\right)_{j-1} \left.\frac{\partial P}{\partial x}\right|_{j-1} = \frac{G_w}{CC1\ \rho_w} \qquad \text{(Boundary Condition)} \qquad (C-16)$$

$$\left|\frac{G_w(\Delta x)}{CC1\ \rho_w}\right|^{(n+1)} + \left(\frac{k_w}{\mu_w}\right)_{j+1/2}^{(n)} (P_{j+1} - P_j)^{(n)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)} \left[S_{o_j}^{(n)} - S_{o_j}^{(n+1)} + C_w(1-S_{o_j}^{(n+1/2)})(P_j^{(n+1)} - P_j^{(n)}) \right] \qquad (C-17)$$

The following relations are applicable in all finite difference formulations in this work.

$$\left(\frac{k}{\mu}\right)_{j-1/2} = \frac{1}{2}\left(\frac{k}{\mu}\right)_j + \frac{1}{2}\left(\frac{k}{\mu}\right)_{j-1} \qquad \text{(C-18)}$$

$$\left(\frac{k}{\mu}\right)_{j+1/2} = \frac{1}{2}\left(\frac{k}{\mu}\right)_j + \frac{1}{2}\left(\frac{k}{\mu}\right)_{j+1} \qquad \text{(C-19)}$$

$$S_{o_j}^{(n+1/2)} = \frac{S_{o_j}^{(n+1)} + S_{o_j}^{(n)}}{2} \qquad \text{(C-20)}$$

GRID POINT (j=2)-REVERSE SWEEP-FIGURE(C-1-f)

Oil

$$\frac{\left(\frac{k_o}{\mu_o}\right)_{j+1/2}^{(n+1)} \left.\frac{\partial P}{\partial x}\right|_{j+1/2}^{(n+1)} - \left(\frac{k_o}{\mu_o}\right)_{j-1}^{(n)} \left.\frac{\partial P}{\partial x}\right|_{j-1}^{(n)}}{\Delta x}$$

$$= \frac{\phi}{CC1(\Delta t)}\left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)}\left(P_j^{(n+1)} - P_j^{(n)}\right)\right] \qquad \text{C-21)}$$

Equation (C-13) is still applicable, therefore equation (C-21) becomes:

$$\left(\frac{k_o}{\mu_o}\right)^{(n+1)}_{j+1/2} (P_{j+1}-P_j)^{(n+1)} = \frac{\phi(\Delta x)^2}{CC1(\Delta t)}\left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)} (P_j^{(n+1)} - P_j^{(n)}\right]$$

(C-22)

Water

$$\left(\frac{k_w}{\mu_w}\right)^{(n+1)}_{j+1/2} \left.\frac{\partial P}{\partial x}\right|^{(n+1)}_{j+1/2} - \left(\frac{k_w}{\mu_w}\right)^{(n)}_{j-1} \left.\frac{\partial P}{\partial x}\right|^{(n)}_{j-1}$$

$$= \frac{\phi}{CC1(\Delta t)}\left[S_{o_j}^{(n)} - S_{o_j}^{(n+1)} + C_w(1-S_{o_j}^{(n+1/2)})(P_j^{(n+1)} - P_j^{(n)})\right] \quad \text{(C-23)}$$

Equation (C-16) is still applicable, therefore equation (C-23) becomes:

$$\left[\frac{G_w(\Delta x)}{CC1\ \rho_w}\right]^{(n)} + \left(\frac{k_w}{\mu_w}\right)^{(n+1)}_{j+1/2} (P_{j+1} - P_j)^{(n+1)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)}\left[S_{o_j}^{(n)} - S_{o_j}^{(n+1)} + C_w(1-S_{o_j}^{(n+1/2)})(P_j^{(n+1)} - P_j^{(n)})\right] \quad \text{(C-24)}$$

INTERIOR GRID POINTS (2<j<(L-Δx/2))-FORWARD SWEEP-FIGURE(C-1-b)

Oil

$$\left(\frac{k_o}{\mu_o}\right)_{j-1/2}^{(n+1)} (P_{j-1}-P_j)^{(n+1)} + \left(\frac{k_o}{\mu_o}\right)_{j+1/2}^{(n)} (P_{j+1}-P_j)^{(n)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)} \left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)} (P_j^{(n+1)} - P_j^{(n)}) \right] \qquad \text{(C-25)}$$

Water

$$\left(\frac{k_w}{\mu_w}\right)_{j-1/2}^{(n+1)} (P_{j-1}-P_j)^{(n+1)} + \left(\frac{k_w}{\mu_w}\right)_{j+1/2}^{(n)} (P_{j+1}-P_j)^{(n)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)} \left[S_{o_j}^{(n)} - S_{o_j}^{(n+1)} + C_w (1-S_{o_j}^{(n+1/2)}) (P_j^{(n+1)} - P_j^{(n)} \right] \qquad \text{(C-26)}$$

INTERIOR GRID POINTS(2<j<(L-Δx/2))-REVERSE SWEEP-FIGURE(C-1-e)

Oil

$$\left(\frac{k_o}{\mu_o}\right)_{j-1/2}^{(n)} (P_{j-1}-P_j)^{(n)} + \left(\frac{k_o}{\mu_o}\right)_{j+1/2}^{(n+1)} (P_{j+1}-P_j)^{(n+1)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)} \left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)} (P_j^{(n+1)} - P_j^{(n)}) \right] \qquad \text{(C-27)}$$

Water

$$\left(\frac{k_w}{\mu_w}\right)^{(n)}_{j-1/2} (P_{j-1}-P_j)^{(n)} + \left(\frac{k_w}{\mu_w}\right)^{(n+1)}_{j+1/2} (P_{j+1}-P_j)^{(n+1)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)} \left[S_{o_j}^{(n)} - S_{o_j}^{(n+1)} + C_w(1-S_{o_j}^{(n+1/2)})(P_j^{(n+1)}-P_j^{(n)}) \right] \quad (C-28)$$

GRID POINT(j=L-Δx/2)-FORWARD SWEEP-FIGURE(C-1-c)

Oil

$$\frac{\left(\frac{k_o}{\mu_o}\right)^{(n)}_{j+1/2} \left.\frac{\partial P}{\partial x}\right|^{(n)}_{j+1/2} - \left(\frac{k_o}{\mu_o}\right)^{(n+1)}_{j-1/2} \left.\frac{\partial P}{\partial x}\right|^{(n+1)}_{j-1/2}}{3/4(\Delta x)}$$

$$= \frac{\phi}{CC1(\Delta t)} \left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)} (P_j^{(n+1)}-P_j^{(n)}) \right] \quad (C-29)$$

$$\frac{8}{3} \left(\frac{k_o}{\mu_o}\right)^{(n)}_{j+1/2} (P_{j+1}-P_j)^{(n)} + \frac{4}{3} \left(\frac{k_o}{\mu_o}\right)^{(n+1)}_{j-1/2} (P_{j-1}-P_j)^{(n+1)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)} \left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)} (P_j^{(n+1)}-P_j^{(n)}) \right] \quad (C-30)$$

Water

$$\frac{8}{3}\left(\frac{k_w}{\mu_w}\right)^{(n)}_{j+1/2}(P_{j+1}-P_j)^{(n)} + \frac{4}{3}\left(\frac{k_o}{\mu_o}\right)^{(n+1)}_{j-1/2}(P_{j-1}-P_j)^{(n+1)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)}\left[S_{o_j}^{(n)} - S_{o_j}^{(n+1)} + C_w(1-S_{o_j}^{(n+1/2)})(P_j^{(n+1)}-P_j^{(n)})\right] \qquad (C-31)$$

GRID POINT (j=L-Δx/2)-REVERSE SWEEP-FIGURE (C-1-d)

Oil

$$\frac{\left(\frac{k_o}{\mu_o}\right)^{(n+1)}_{j+1/2}\left.\frac{\partial P}{\partial x}\right|^{(n+1)}_{j+1/2} - \left(\frac{k_o}{\mu_o}\right)^{(n)}_{j-1/2}\left.\frac{\partial P}{\partial x}\right|^{(n)}_{j-1/2}}{3/4(\Delta x)}$$

$$= \frac{\phi}{CC1(\Delta t)}\left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)}(P_j^{(n+1)} - P_j^{(n)})\right] \qquad (C-32)$$

$$\frac{8}{3}\left(\frac{k_o}{\mu_o}\right)^{(n+1)}_{j+1/2}(P_{j+1}-P_j)^{(n+1)} + \frac{4}{3}\left(\frac{k_o}{\mu_o}\right)^{(n)}_{j-1/2}(P_{j-1}-P_j)^{(n)}$$

$$= \frac{\phi(\Delta x)^2}{CC1(\Delta t)}\left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)}(P_j^{(n+1)} - P_j^{(n)})\right] \qquad (C-33)$$

Water

$$\frac{8}{3}\left(\frac{k_w}{\mu_w}\right)_{j+1/2}^{(n+1)}(P_{j+1}-P_j)^{(n+1)}+\frac{4}{3}\left(\frac{k_w}{\mu_w}\right)_{j-1/2}^{(n)}(P_{j-1}-P_j)^{(n)}$$

$$=\frac{\phi(\Delta x)^2}{CC1(\Delta t)}\left[S_{o_j}^{(n)}-S_{o_j}^{(n+1)}+C_w(1-S_{o_j}^{(n+1/2)})(P_j^{(n+1)}-P_j^{(n)})\right] \quad (C-34)$$

Newton-Raphson Iterative Procedure

If the solution to the problem is started at the first grid point beyond the inlet boundary, the only unknown variables in the finite difference formulations are $S_{o_j}^{(n+1)}$ and $P_j^{(n+1)}$ and any fluid properties which are functions of these two parameters. Since the fluid properties are dependent on the two variables an iterative technique will be required to solve the equations simultaneously. The Newton-Raphson iterative procedure should suffice. Although the method does not guarantee convergence, especially for non-linear problems, no difficulty is expected as there are only two variables and an initial guess of these two parameters can be made reasonably close to the physical

problem.

In general terms, if $\underline{x}^{(0)}$ is the initial guess of the solution then $\underline{F}$ may be expanded about $\underline{x}^{(0)}$.

$$\underline{F}(\underline{x}) = \underline{F}(\underline{x}^{(0)}) + \underline{J}\ (\underline{x}^{(0)})\ [\ \underline{x} - \underline{x}^{(0)}] \qquad \text{(C-35)}$$

where $\underline{J}\ (\underline{x})$ is defined to be the Jacobian Matrix,

$$\underline{J}\ (\underline{x}) = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} \end{bmatrix} \qquad \text{(C-36)}$$

The solution of equation (C-35) will be the next, supposedly improved, guess.

$$\underline{x}^{(1)} = \underline{x}^{(0)} - \underline{J}^{-1}\ (\underline{x}^{(0)})\ \underline{F}\ (\underline{x}^{(0)}) \qquad \text{(C-37)}$$

A recurrence relation for the (r+1) iteration may be formulated as follows;

$$\underline{x}^{(r+1)} = \underline{x}^{(r)} - \underline{J}^{-1}(\underline{x}^{(r)})\ \underline{F}\ (\underline{x}^{(r)}) \qquad \text{(C-38)}$$

and termination of the iteration, or convergence, is executed when

$$\left| x_j^{(r+1)} - x_j^{(r)} \right| \leq \varepsilon_j \qquad j=1,2 \tag{C-39}$$

where ε_j is specified sufficiently small to satisfy the problems's required accuracy.

It is easily seen that the inversion of the Jacobian Matrix will dictate the ease of solving the problem by this technique. The problem, with only two variables, will require the solution of a 2 x 2 matrix and this is very straightforward and should offer no difficulty.

To be more specific, if the iteration variables are denoted by $P_K = P_j^{(n+1)}$ and $S_{OK} = S_{o_j}^{(n+1)}$ then the continuity-flow equations for the oil and water phase may be written as;

$$\text{Oil Phase} \qquad \phi\ (P_K, S_{OK}) = 0 \tag{C-40}$$

$$\text{Water Phase} \qquad \psi\ (P_K, S_{OK}) = 0 \tag{C-41}$$

where ϕ and ψ are functions.

If an initial guess for P_K and S_{OK} is made and if y_1 and y_2 are the successive corrections to be made to the guesses, then

$$\phi(P_K+y_1, S_{OK}+y_2) = \phi(P_K, S_{OK}) + y_1 \frac{\partial\phi(P_K, S_{OK})}{\partial P_K} + y_2 \frac{\partial\phi(P_K, S_{OK})}{\partial S_{OK}}$$

$$= 0 \qquad (C-42)$$

and similarly,

$$\psi(P_K+y_1, S_{OK}+y_2) = \psi(P_K, S_{OK}) + y_1 \frac{\partial\psi(P_K, S_{OK})}{\partial P_K} + y_2 \frac{\partial\psi(P_K, S_{OK})}{\partial S_{OK}}$$

$$= 0 \qquad (C-43)$$

It should be noted that the method is first order correct in that all second order and higher terms are truncated. Upon further simplifications by making the following definitions,

$$\phi_{P_K} = \frac{\partial\phi(P_K, S_{OK})}{\partial P_K} \qquad \psi_{P_K} = \frac{\partial\psi(P_K, S_{OK})}{\partial P_K}$$

$$\phi_{S_{OK}} = \frac{\partial\phi(P_K, S_{OK})}{\partial S_{OK}} \qquad \psi_{S_{OK}} = \frac{\partial\psi(P_K, S_{OK})}{\partial S_{OK}} \qquad (C-44)$$

equations (C-42) and (C-43) may be written as,

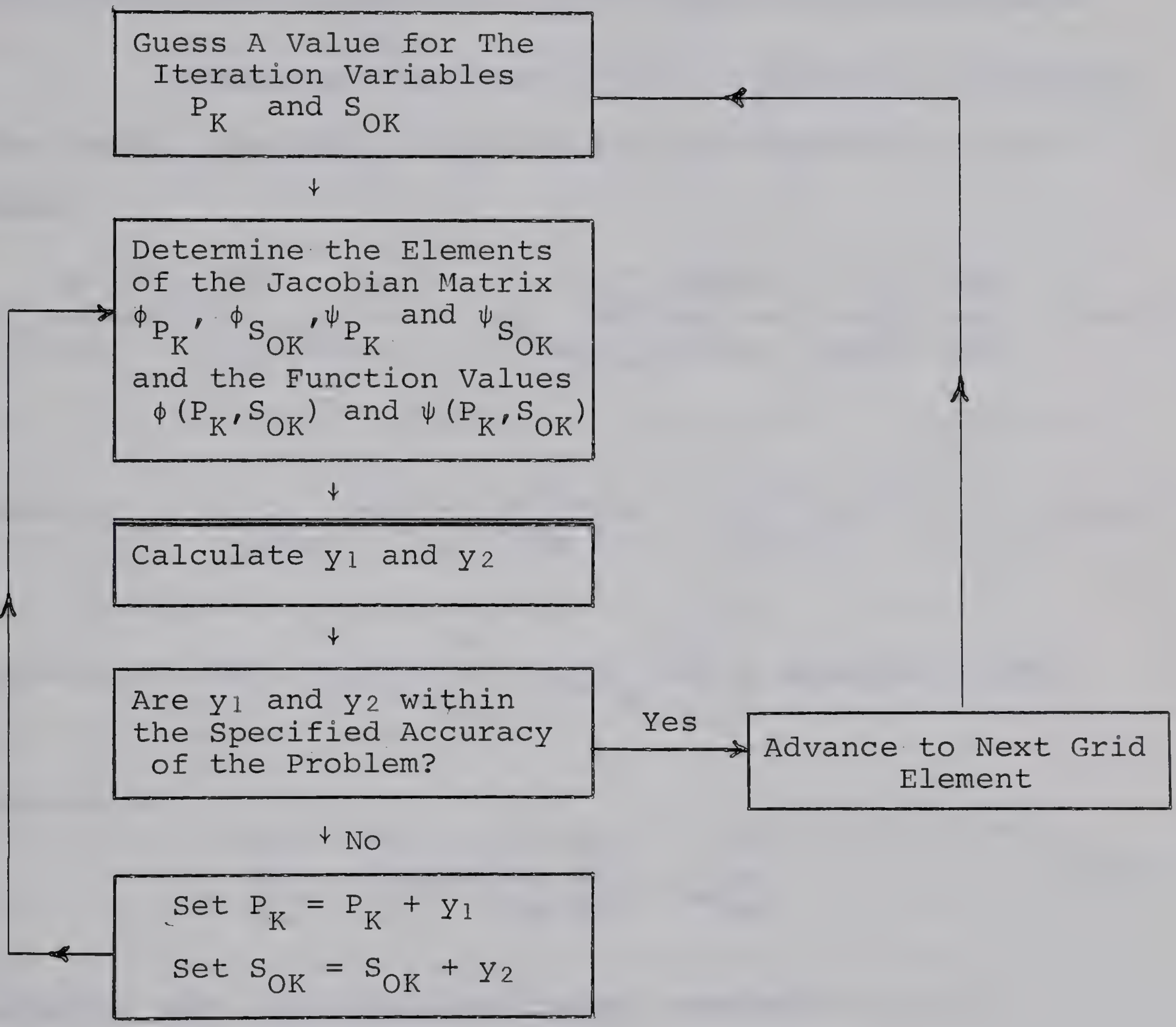
$$\begin{bmatrix} \phi_{P_K} & \phi_{S_{OK}} \\ \psi_{P_K} & \psi_{S_{OK}} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = - \begin{bmatrix} \phi(P_K, S_{OK}) \\ \psi(P_K, S_{OK}) \end{bmatrix} \tag{C-45}$$

Solving equation (C-45) for y_1 and y_2 is straight-forward.

$$y_1 = \frac{\psi(P_K, S_{OK})\ \phi_{S_{OK}} - \phi(P_K, S_{OK})\ \psi_{S_{OK}}}{\psi_{S_{OK}}\ \phi_{P_K} - \psi_{P_K}\ \phi_{S_{OK}}} \tag{C-46}$$

$$y_2 = \frac{\psi(P_K, S_{OK})\ \phi_{P_K} - \phi(P_K, S_{OK})\ \psi_{P_K}}{\psi_{P_K}\ \phi_{S_{OK}} - \phi_{P_K}\ \psi_{S_{OK}}} \tag{C-47}$$

In summary, the Newton-Raphson iterative procedure was executed in the following manner:


Guess A Value for The Iteration Variables P_K and S_{OK}
Determine the Elements of the Jacobian Matrix ϕ_{P_K}, $\phi_{S_{OK}}$, ψ_{P_K} and $\psi_{S_{OK}}$ and the Function Values $\phi(P_K, S_{OK})$ and $\psi(P_K, S_{OK})$
Calculate y_1 and y_2
Are y_1 and y_2 within the Specified Accuracy of the Problem?
Yes
Advance to Next Grid Element
No
Set $P_K = P_K + y_1$
Set $S_{OK} = S_{OK} + y_2$

The Thermal Energy Equation - Crank-Nicolson Approximation

Equations (B-20) and (B-21) in Appendix B describe the thermal phenomena of conduction and convection in the model.

$$\rho C_p \frac{\partial T}{\partial t} = CC1 \left\{ \left[\rho_o C_{p_o} \frac{k_o}{\mu_o} + \rho_w C_{p_w} \frac{k_w}{\mu_w} \right] \frac{\partial P}{\partial x} \right\} \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(k_e \frac{\partial T}{\partial x} \right) \qquad \text{(C-48)}$$

$$\text{where } \rho C_p = [\rho_w C_{p_w} (1-S_o) + \rho_o C_{p_o} S_o] \phi + \rho_r C_{p_r} (1-\phi) \qquad \text{(C-49)}$$

By defining $ACV = \left[\rho_o C_{p_o} \frac{k_o}{\mu_o} + \rho_w C_{p_w} \frac{k_w}{\mu_w} \right]$, equation (C-48)

reduces to,

$$\rho C_p \frac{\partial T}{\partial t} = CC1(ACV) \frac{\partial P}{\partial x} \frac{\partial T}{\partial x} + k_e \frac{\partial^2 T}{\partial x^2} \qquad \text{(C-50)}$$

assuming that the effective thermal conductivity (k_e) is a function of saturation only.

The boundary conditions and the initial condition for temperature [T(x,t)], were specified as:

$$T(0,t) = T_{INJ}$$

$$\left.\frac{\partial T}{\partial t}\right|_{x=L} = 0.0 \qquad \text{(C-51)}$$

$$T(x,0) = T_i$$

Using the Crank-Nicolson procedure, the finite difference approximation to equation (C-50) will be implicit and hence a series of equations must be developed across the grid network of Figure (C-1) until the outlet boundary is reached. Since the outlet boundary condition is specified the problem may now be solved. The solution involves the inversion of a matrix and it is assumed that this inverse exists.

Equation (C-50) in discretized form using equations (C-7) and (C-8) to approximate the derivatives may be used to describe the inlet boundary point and interior grid points.

$$\frac{\partial T}{\partial t} = \frac{CC1(ACV)}{\rho C_p}\left[\frac{-P_{j+2}+6P_{j+1}-3P_j-2P_{j-1}}{6(\Delta x)}\right]\left[\frac{-T_{j+2}+6T_{j+1}-3T_j-2T_{j-1}}{6(\Delta x)}\right]$$

$$+\frac{k_e}{\rho C_p}\left[\frac{T_{j+1}-2T_j+T_{j-1}}{(\Delta x)^2}\right] \qquad \text{(C-52)}$$

Forms of equations (C-7) and (C-9) to approximate the derivatives are required to describe the last grid point before the outlet boundary.

The discretized form of the equation is:

$$\frac{\partial T}{\partial t} = \frac{CC1(ACV)}{\rho C_p}\left[\frac{2P_{j+1}+3P_j-6P_{j-1}+P_{j-2}}{6(\Delta x)}\right]\left[\frac{2T_{j+1}+3T_j-6T_{j-1}+T_{j-2}}{6(\Delta x)}\right]$$

$$+ \frac{k_e}{\rho C_p}\left[\frac{T_{j+1}-2T_j+T_{j-1}}{(\Delta x)^2}\right] \qquad \text{(C-53)}$$

Applying the Crank-Nicolson procedure to equation (C-52) results in:

$$\left[\frac{T_j^{(n+1)} - T_j^{(n)}}{\Delta t}\right]$$

$$= \left[\frac{CC1(ACV)}{36(\Delta x)^2(\rho C_p)}\right]^{(n)}\left[-P_{j+2}+6P_{j+1}-3P_j-2P_{j-1}\right]^{(n)}\left\{\frac{1}{2}\left[-T_{j+2}+6T_{j+1}-3T_j-2P_{j-1}\right]^{(n)}\right.$$

$$\left. + \frac{1}{2}\left[-T_{j+2}+6T_{j+1}-3T_j-2T_{j-1}\right]^{(n+1)}\right\}$$

$$+ \left[\frac{k_e}{\rho C_p(\Delta x)^2}\right]^{(n)}\left\{\frac{1}{2}\left[T_{j+1}-2T_j+T_{j-1}\right]^{(n)} + \frac{1}{2}\left[T_{j+1}-2T_j+T_{j-1}\right]^{(n+1)}\right\} \qquad \text{C-54)}$$

Rearranging equation (C-54) so that terms to be evaluated at time (n+1) are on one side of the equation and terms to be evaluated at time (n) are on the other side of the equation, the final formulation of the finite difference approximation to equation (C-50) for the inlet boundary and interior grid points is developed.

$$
\begin{aligned}
&T_j^{(n+1)}\left\{\frac{2\rho C_p(\Delta x)^2}{CC1(ACV)\Delta t}+\frac{1}{12}\left[-P_{j+2}+6P_{j+1}-3P_j-2P_{j-1}\right]+\frac{2k_e}{CC1(ACV)}\right\}^{(n)}\\
&-T_{j+1}^{(n+1)}\left\{\frac{1}{6}\left[-P_{j+2}+6P_{j+1}-3P_j-2P_{j-1}\right]+\frac{k_e}{CC1(ACV)}\right\}^{(n)}\\
&+T_{j+2}^{(n+1)}\left\{\frac{1}{36}\left[-P_{j+2}+6P_{j+1}-3P_j-2P_{j-1}\right]\right\}^{(n)}\\
&+T_{j-1}^{(n+1)}\left\{\frac{1}{18}\left[-P_{j+2}+6P_{j+1}-3P_j-2P_{j-1}\right]-\frac{k_e}{CC1(ACV)}\right\}^{(n)}=\frac{2\rho C_p(\Delta x)^2}{CC1(ACV)\Delta t}\,T_j^{(n)}
\end{aligned}
$$

$$+\left\{\frac{1}{36}\left[-P_{j+2}+6P_{j+1}-3P_j-2P_{j-1}\right]^{(n)}\left[-T_{j+2}+6T_{j+1}-3T_j-2T_{j-1}\right]^{(n)}\right\}$$

$$+\left\{\frac{k_e}{CC1(ACV)}\left(T_{j+1}-2T_j+T_{j-1}\right)\right\}^{(n)} \qquad \text{(C-55)}$$

Similarly, applying the Crank-Nicolson procedure to equation (C-53) results in:

$$\left[\frac{T_j^{(n+1)}-T_j^{(n)}}{\Delta t}\right]$$

$$=\left[\frac{(CC1)\ ACV}{36(\Delta x)^2\rho C_p}\right]^{(n)}\left[2P_{j+1}+3P_j-6P_{j-1}+P_{j-2}\right]^{(n)}\left\{\frac{1}{2}\left[2T_{j+1}+3T_j-6T_{j-1}+T_{j-2}\right]^{(n)}\right.$$

$$\left.+\frac{1}{2}\left[2T_{j+1}+3T_j-6T_{j-1}+T_{j-2}\right]^{(n+1)}\right\}+\left[\frac{k_e}{\rho C_p(\Delta x)^2}\right]^{(n)}\left\{\frac{1}{2}\left[T_{j+1}-2T_j+T_{j-1}\right]^{(n)}\right.$$

$$\left.+\frac{1}{2}\left[T_{j+1}-2T_j+T_{j-1}\right]^{(n+1)}\right\} \qquad \text{C-56)}$$

Rearranging equation (C-56) in a like manner to equation (C-55) the final formulation of the finite difference approximation to equation (C-50) for the outlet end of the model is developed.

$$T_j^{(n+1)}\left\{\frac{2\rho C_p(\Delta x)^2}{CC1(ACV)\Delta t} - \frac{1}{12}\left[2P_{j+1}+3P_j-6P_{j-1}+P_{j-2}\right] + \frac{2k_e}{CC1(ACV)}\right\}^{(n)}$$

$$-T_{j+1}^{(n+1)}\left\{\frac{1}{18}\left[2P_{j+1}+3P_j-6P_{j-1}+P_{j-2}\right] + \frac{k_e}{(CC1)(ACV)}\right\}^{(n)}$$

$$-T_{j-2}\left\{\frac{1}{36}\left[2P_{j+1}+3P_j-6P_{j-1}+P_{j-2}\right]\right\}^{(n)}$$

$$+T_{j-1}^{(n+1)}\left\{\frac{1}{6}\left[2P_{j+1}+3P_j-6P_{j-1}+P_{j-2}\right] - \frac{k_e}{CC1(ACV)}\right\}^{(n)} = \frac{2\rho C_p(\Delta x)^2}{CC1(ACV)\Delta t}T_j^{(n)}$$

$$+\left\{\frac{1}{36}\left[2P_{j+1}+3P_j-6P_{j-1}+P_{j-2}\right]^{(n)}\left[2T_{j+1}+3T_j-6T_{j-1}+T_{j-2}\right]^{(n)}\right\}$$

$$+\left\{\frac{k_e}{CC1(ACV)}\left(T_{j+1}-2T_j+T_{j-1}\right)\right\}^{(n)} \qquad (C-57)$$

The use of third order correct approximations for the derivatives results in the formation of a non-symmetric matrix of equations. The square root method, extended to non-symmetric matrices, was employed to solve the problem.

APPENDIX D

COMPARISON OF THE "OPEN-FLOW" AND "NO-FLOW" BOUNDARY PROBLEMS

Comparison of the "Open-Flow" and "No-Flow" Boundary Problems

Figure 3 illustrated two linear models: (a)"no-flow" boundary case and, (b) "open-flow" boundary case. The primary difference in the mathematical treatment of the two models for an injection or production problem lies in the specification of the boundary conditions. For model (a), there is no flow across the inlet and outlet boundaries. A source term is required to describe the injection into or production from the model but without the necessity of specifying where the source term is located. Model (b) on the other hand specifies that flow may occur across the inlet and outlet boundaries and hence fluid may be injected or produced without specifying a source term. Model (b) was discussed in the section of "Theory and Literature Review - Hot-Water Injection - Two Phase Flow" and was developed fully in Appendix B.

Model (a) will be developed in this section. The continuity-Darcy flow equations may be written for each phase with the mass injection rate of water being represented by a source term.

Oil

$$\frac{\partial}{\partial x}\left(\frac{\rho_o k_o h}{\mu_o}\frac{\partial P}{\partial x}\right) dxdy + W_o hdy = \phi hdxdy \frac{\partial}{\partial t}(\rho_o S_o) \qquad \text{(D-1)}$$

Water

$$\frac{\partial}{\partial x}\left(\frac{\rho_w k_w h}{\mu_w}\frac{\partial P}{\partial x}\right) dxdy + W_w hdy = \phi hdxdy \frac{\partial}{\partial t}(\rho_w S_w) \qquad \text{(D-2)}$$

where W_o = mass rate input of oil = 0

W_w = mass rate input of water $= \left(\frac{\rho_w k_w}{\mu_w}\frac{\partial P}{\partial x}\right)_{x=0}$

= CONSTANT (at injection point)

= 0 (at all other grid points)

Since the reservoir thickness, h, is not a function of x for this problem and the mass rate input of oil is zero equations (D-1) and (D-2) may be written as:

$$\frac{\partial}{\partial x}\left(\frac{\rho_o k_o}{\mu_o}\frac{\partial P}{\partial x}\right) dxdy = \phi dxdy \frac{\partial}{\partial t}(\rho_o S_o) \qquad \text{(D-3)}$$

$$\frac{\partial}{\partial x}\left(\frac{\rho_w k_w}{\mu_w}\frac{\partial P}{\partial x}\right) dxdy + W_w dy = \phi dxdy \frac{\partial}{\partial t}(\rho_w S_w) \qquad \text{(D-4)}$$

Equations (D-3) and (D-4) may be manipulated in the same manner as the hydrodynamical equations (B-1) and (B-2) were in Appendix B giving rise to equations (D-5) and (D-6).

$$\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) = \phi\left[\frac{\partial S_o}{\partial t} + C_o S_o \frac{\partial P}{\partial t}\right] \tag{D-5}$$

$$\frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w}\frac{\partial P}{\partial x}\right) + \frac{W_w}{\rho_w dx} = \phi\left[\frac{\partial(-S_o)}{\partial t} + C_w(1-S_o)\frac{\partial P}{\partial t}\right] \tag{D-6}$$

Equations (B-11) and (B-13) in Appendix B are the analogous hydrodynamical equations for model (b), the "open-flow" boundary case.

$$\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CCl}\left[\frac{\partial S_o}{\partial t} + C_o S_o \frac{\partial P}{\partial t}\right] \tag{B-11}$$

$$\frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w}\frac{\partial P}{\partial x}\right) = \frac{\phi}{CCl}\left[\frac{\partial}{\partial t}(-S_o) + C_w(1-S_o)\frac{\partial P}{\partial t}\right] \tag{B-13}$$

The only difference in the two sets of equations is in the inclusion of a source term $\left(\frac{W_w}{\rho_w dx}\right)$ in equation (D-6) to represent an input of water to the system.

The source term will be equal to zero for interior grid elements and the two sets of equations become identical.

Equations (D-5) and (D-6) will be discretized for the inlet boundary (j=2, Figure C-1). The approximations for the derivatives will be the same as those used in Appendix C and the one-dimensional ADEP will be applied.

Oil Phase (Forward Sweep)

$$\frac{\left(\frac{k_o}{\mu_o}\right)^{(n)}_{j+1/2} \left.\frac{\partial P}{\partial x}\right|^{(n)}_{j+1/2} - \left(\frac{k_o}{\mu_o}\right)^{(n+1)}_{j-1} \left.\frac{\partial P}{\partial x}\right|^{(n+1)}_{j-1}}{(\Delta x)} = \phi\left[\frac{\partial S_o}{\partial t} + C_o S_o \frac{\partial P}{\partial t}\right] \qquad \text{(D-7)}$$

$$-\left(\frac{k_o}{\mu_o}\right)_{j-1} \left.\frac{\partial P}{\partial x}\right|_{j-1} = \frac{W_o}{\rho_o} = 0.0 \text{ (Boundary Condition)} \qquad \text{(D-8)}$$

$$\left(\frac{k_o}{\mu_o}\right)^{(n)}_{j+1/2} (P_{j+1}-P_j)^n = \frac{\phi(\Delta x)^2}{\Delta t}\left[S_{o_j}^{(n+1)} - S_{o_j}^{(n)} + C_o S_{o_j}^{(n+1/2)} (P_j^{(n+1)} - P_j^{(n)})\right] \qquad \text{(D-9)}$$

Equation (D-9) is identical to equation (C-14) in Appendix C which shows that the fluid flow equation for the oil

phase in finite difference form is the same for both model (a) and model (b).

Water Phase (Forward Sweep)

$$\frac{\left(\frac{k_w}{\mu_w}\right)^{(n)}_{j+1/2} \left.\frac{\partial P}{\partial x}\right|^{(n)}_{j+1/2} - \left(\frac{k_w}{\mu_w}\right)^{(n+1)}_{j-1} \left.\frac{\partial P}{\partial x}\right|^{(n+1)}_{j-1}}{(\Delta x)} + \frac{W_w}{\rho_w (\Delta x)}$$

$$= \phi \left[\frac{\partial(-S_o)}{\partial t} + C_w\ (1-S_o)\ \frac{\partial P}{\partial t}\right] \qquad \text{(D-10)}$$

$$\left.\frac{\partial P}{\partial x}\right|^{(n+1)}_{j-1} = 0.0 \text{ (no flow boundary condition)}$$

$$\left[\frac{W_w(\Delta x)}{\rho_w}\right]^{(n+1)} + \left(\frac{k_w}{\mu_w}\right)^{(n)}_{j+1/2} (P_{j+1} - P_j)^{(n)}$$

$$= \frac{\phi(\Delta x)^2}{\Delta t} \left[S_{o_j}^{(n)} - S_{o_j}^{(n+1)} + C_w (1 - S_{o_j}^{(n+1/2)})(P_j^{(n+1)} - P_j^{(n)})\right] \qquad \text{(D-11)}$$

Equation (D-11) is identical to equation (C-17) in Appendix C which shows that the hydrodynamic equation

for the water phase in finite difference form is the same for both model (a) and model (b).

In summary, although the mathematical development of the partial differential equations to describe the hydrodynamical aspect of the problem is different, as shown by a comparison of equations (D-5) and (D-6) for model (a) with equations (B-11) and (B-13) for model (b), the resulting finite difference formulations are identical as shown by a comparison of equations (D-19) and (D-11) for model (a) with equations (C-14) and (C-17) for model (b). This suggests that the solution to the problem will satisfy either the "no-flow" boundary case or the "open-flow" boundary case if the discretization of the partial differential equations is carried out as described.

APPENDIX E

(a) Discussion of Validity of Darcy's Law for Non-Isothermal Systems

Darcy's Law for a newtonian liquid of constant density or of density as a function of pressure only may be written as

$$\vec{q} = \sigma \nabla \Phi$$

where $\sigma = \frac{\rho k}{\mu}$

and $\Phi = gz - \frac{P}{\rho} = \text{POTENTIAL}$

Since Φ is a scalar potential and $\vec{q}$ is a flux vector it is essential that $\vec{q}$ be irrotational. This implies that the circulation (Γ) must equal zero, where

$$\Gamma = \oint \vec{q} \circ d\vec{s} = 0$$

$\vec{s}$ - refers to a surface

Hubbert(1) indicates that this equation will be valid for isothermal systems. Davidson(2) shows for nonisothermal reservoirs the circulation is not zero and hence a rotational flow field develops. Strictly speaking it appears

Darcy's law is not valid for nonisothermal systems. However, the magnitude of the deviation in flow behavior if specified by Darcy's Law under nonisothermal conditions is unknown. Lacking an alternative, this work assumes the effect of nonisothermal conditions on Darcy's Law to be negligible.

(b) Validating the Use of an Effective Thermal Conductivity in The Thermal Energy Equation

The use of an effective thermal conductivity appears to be an acceptable approach in describing heat transfer problems using Fourier's law. Bailey and Larkin(3) demonstrate the assumption of instantaneous heat transfer between the solid and liquid phases of the porous system is quite reasonable. This work therefore assumes the use of an effective thermal conductivity is valid and hence the temperature of the reservoir rock and its surrounding fluid are the same.

(1) HUBBERT, M.K.: "The Theory of Ground-Water Motion", Jour. of Geol., 48, (1940), p. 785.

(2) DAVIDSON, L.B.: "A Theoretical Analysis of Reservoir Behavior During the Intermittent Injection of Steam", Ph.D. Thesis, Stanford University, March, 1966.

(3) BAILEY, H.R. and LARKIN, B.K.:"Conduction-Convection in Underground Combustion", Trans.AIME, 219(1960), p. 320.

APPENDIX F

RESERVOIR AND FLUID PROPERTIES - COMPUTER INPUT PARAMETERS

RESERVOIR AND FLUID PROPERTIES - COMPUTER INPUT PARAMETERS

ϕ = Porosity = 0.24

ρ_r = Rock Density = 165.2 lbm/ft^3

ρ_o^* = Oil Density at (P*=14.7 psia) = 55.0 lbm/ft^3

ρ_w^* = Water Density at (P*=14.7 psia) = 62.4 lbm/ft^3

μ_o^* = Oil Viscosity at 60°F = 265 cp.

C_{p_r} = Specific Heat of Rock = 0.23 BTU/lbm°F

C_{p_w} = Specific Heat of Water = 1.0 BTU/lbm°F

C_{p_o} = Specific Heat of Oil = 0.650 BTU/lbm°F

K = Absolute Permeability = 1000 md

k_e^* = Effective Thermal Conductivity = 0.5 BTU/hr.ft°F

C_o = Oil Compressibility = $3.0(10^{-5})$ psia^{-1}

C_w = Water Compressibility = $3.2(10^{-6})$ psia^{-1}

S_{wc} = Connate Water Saturation = 0.20

S_{wi} = Initial Water Saturation = 0.25

S_{oi} = Initial Oil Saturation = 0.75

P_i = Initial Pressure Distribution = Constant = 14.7 psia

T_i = Initial Temperature Distribution = Constant = 60°F

T_{INJ} = Water Injection Temperature = 200°F

FUNCTION DEFINITIONS

Oil Density $$\rho_o(P) = \rho_o^* \; e^{C_o(P-P^*)}$$

Water Density $$\rho_w(P) = \rho_w^* \; e^{C_w(P-P^*)}$$

Effective Permeability to Oil (Figure F-1)

$$k_o(S_o) = K \left[1 - \left(\frac{1-S_o-S_{wc}}{1-S_{wc}} \right) \right]^2 \left[1 - \left(\frac{1-S_o-S_{wc}}{1-S_{wc}} \right)^2 \right] \quad (1)$$

$$0.0 \leq S_o \leq 0.8$$

Effective Permeability to Water (Figure F-1)

$$k_w(S_o) = K \left[\frac{1-S_o-S_{wc}}{1-S_{wc}} \right]^4 \qquad 0.0 \leq S_o \leq 0.8 \qquad (1)$$

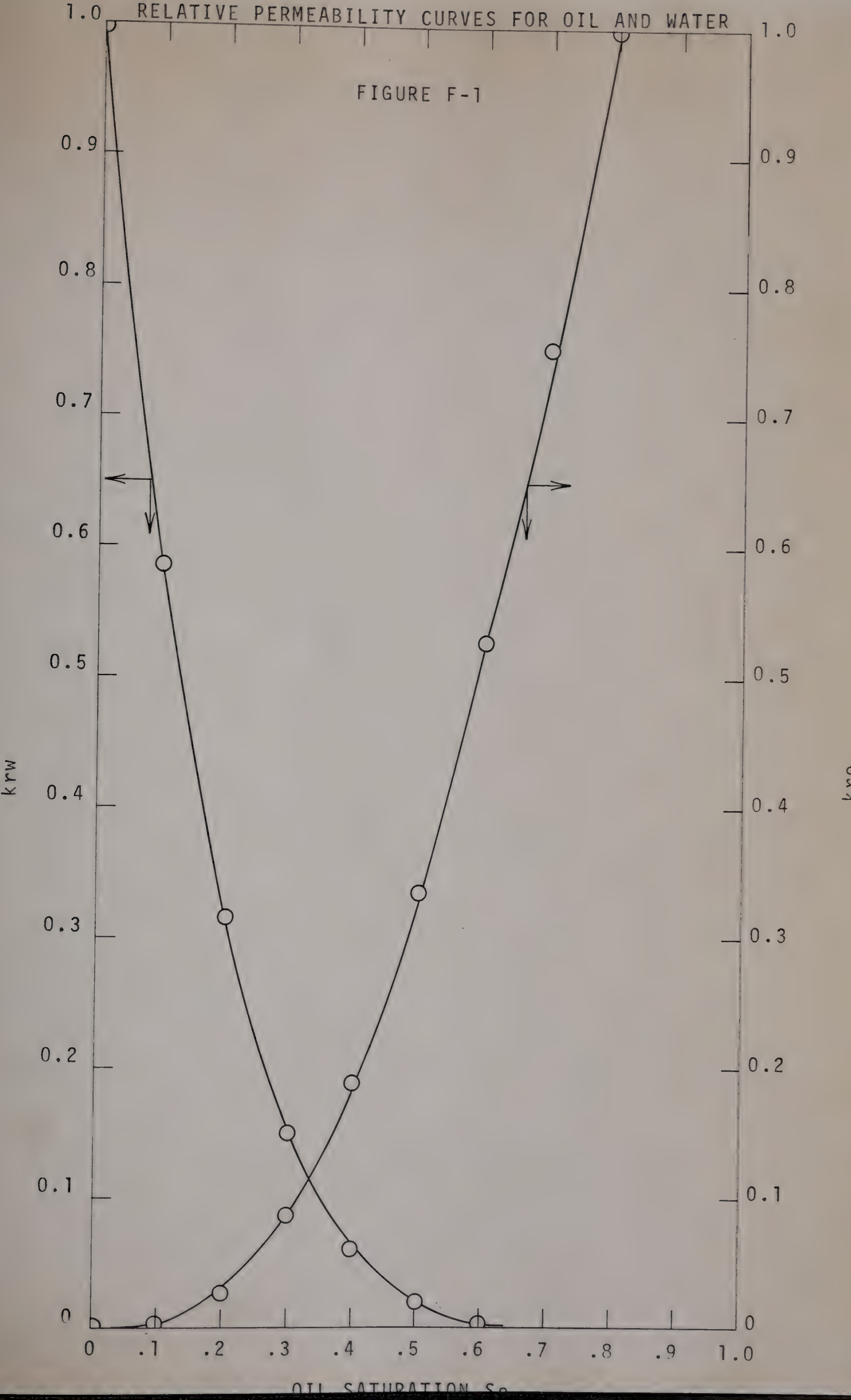
Water Viscosity

$$\mu_w(T) = \left[\frac{1776-T}{2.65(T)-89} \right] \qquad \text{(Figure F-2)} \qquad (2)$$

Oil Viscosity

$$\mu_o(T) = \mu_o^* \; (60)^{\surd} \; T^{-\surd} \qquad \text{(Figure F-3)} \qquad (2)$$

where the value of (√) determines the sensitivity of the oil viscosity to temperature changes.

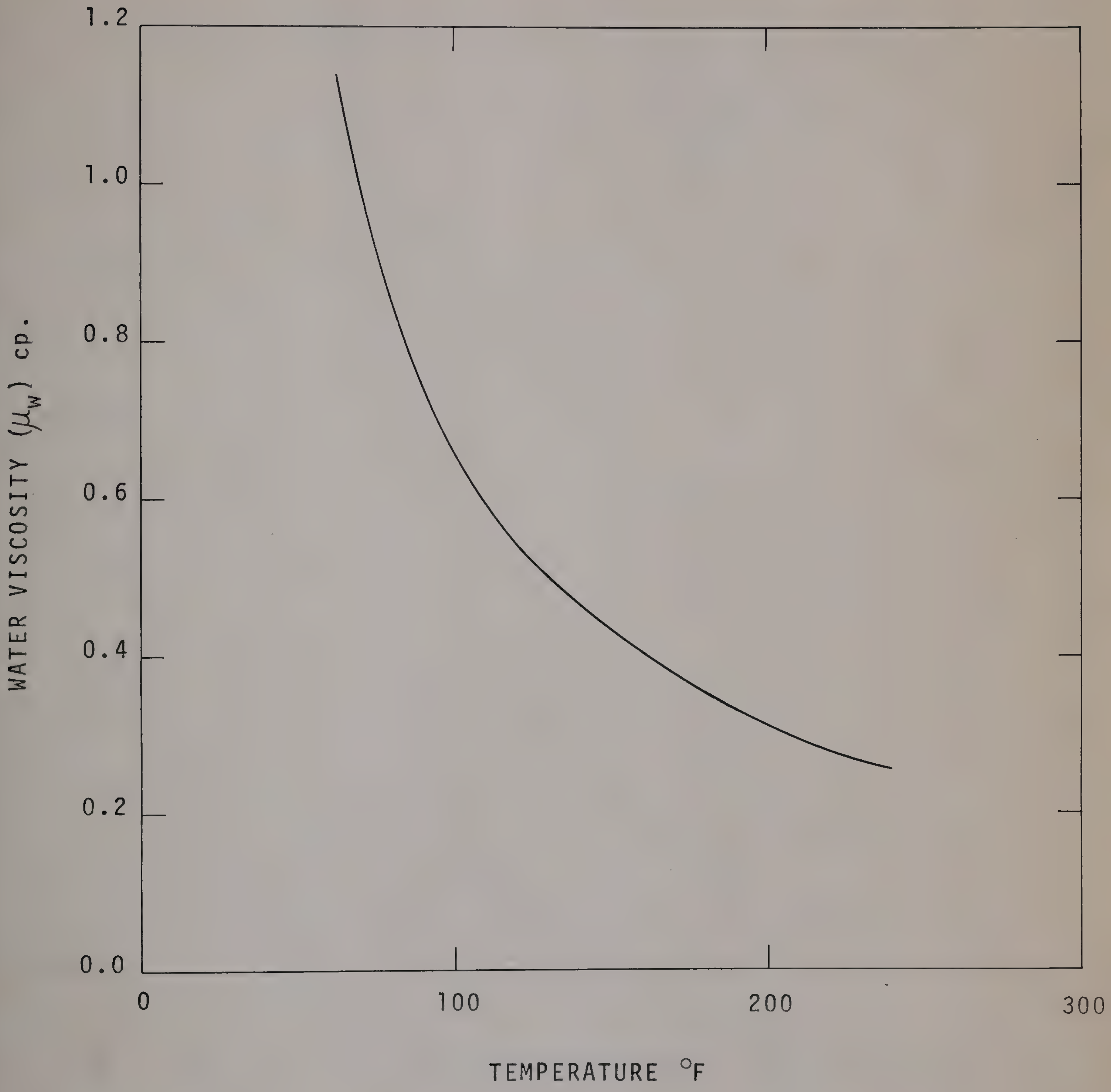


RELATIVE PERMEABILITY CURVES FOR OIL AND WATER

FIGURE F-1

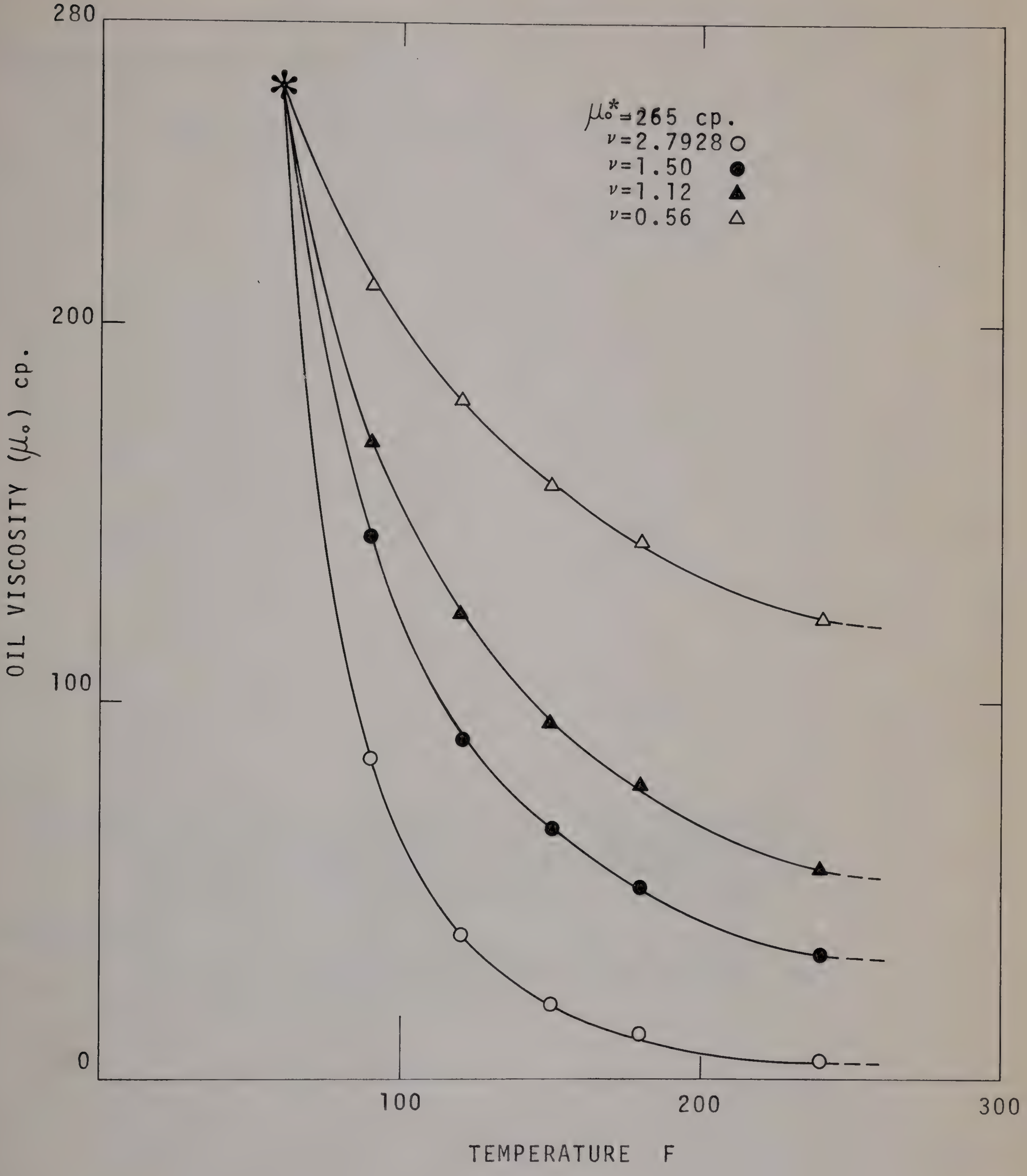
WATER VISCOSITY VERSUS TEMPERATURE

FIGURE F-2



OIL VISCOSITY VERSUS TEMPERATURE

FIGURE F-3



Effective Thermal Conductivity

$$k_e(S_o) = k_e^* (1 + S_o)$$

---

(1) WYLLIE, M.R.J. and GARDNER, G.H.F.: "The Generalized Kozeny-Carman Equation -- Its Application to Problems of Multiphase Flow in Porous Media", World Oil, Production Section, April 1958, p. 225.

(2) GOTTFRIED, B.S.: "A Mathematical Model of Thermal Oil Recovery in Linear Systems", Soc. Petrol. Engr. J., Sept. 1965, p. 210

APPENDIX G

THE COMPUTER PROGRAM

THE COMPUTER PROGRAM

A computer program was written to solve the hot-water injection problem. The program may be divided into four parts.

(1) MAIN PROGRAM - The program will compute the pressure and saturation distribution in a linear, one-dimensional, homogeneous reservoir model. This is accomplished by solving the continuity-Darcy flow equations for oil and water (Equations (11) and (12)) simultaneously, using the alternating direction explicit procedure (written for one-dimension) and the Newton-Raphson iterative technique.

(2) MATERIAL BALANCE SUBROUTINE - A cumulative and instantaneous material balance is carried out for each phase (oil and water) on a mass basis as a check on the main program results. Deviations from 1.0 will indicate the fractional error in the material balance.

(3) MATRIX FORMULATION FOR TEMPERATURE DISTRIBUTION - The pressure and saturation distributions obtained from the main program results are used to solve

the thermal energy equation (15). An equation must be written for every grid element since the equation is implicit. This subroutine forms the matrix of thermal energy equations (Crank-Nicolson Finite Difference Approximations) which must be solved to obtain the desired temperature distribution at the end of the time interval.

(4) INVERSION OF A NON-SYMMETRIC MATRIX BY A MODIFIED SQUARE ROOT METHOD - This subroutine inverts the matrix which was developed by the subroutine discussed in Part (3). The temperature distribution is then calculated and used in the MAIN PROGRAM to calculate the temperature dependent properties for the next time interval. Mathematically, this subroutine performs the following calculations. If the thermal energy equations written for each grid element may be represented in matrix notation as,

$$\underline{A}\ \underline{T} = \underline{b}$$

the program will first of all find the inverse of $\underline{A}$ (assuming an inverse exists), then

$$\underline{A}^{-1}\underline{A}\ \underline{T} = \underline{A}^{-1}\ \underline{b}$$

or

$$\underline{I}\ \underline{T} = \underline{A}^{-1}\ \underline{b}$$

or

$$\underline{T} = \underline{A}^{-1}\ \underline{b}$$

The solution of $\underline{T} = \underline{A}^{-1}\underline{b}$ will yield the desired temperature distribution.

COMPUTER PROGRAM NOMENCLATURE

CALL MATBAL = call material balance subroutine

CALL SOLUT = call subroutine SOLUT to invert the matrix

CALL TEMP = call the temperature subroutine to set up the matrix

CC1 = CC1 = field unit conversion constant

CO = C_o = oil compressibility

CPO(T) = function definition for specific heat of oil at constant pressure

CPOSTR=$C^*_{p_o}$ = specific heat of oil at constant pressure

CPW(T) = function definition for specific heat of water at constant pressure

CPWSTR=$C^*_{p_w}$ = specific heat of water at constant pressure

CPROCK=C_{p_r} = specific heat of reservoir rock at constant pressure

CUMMBO = cumulative material balance of oil

CUMMBW = cumulative material balance of water

CW=C_w = water compressibility

D8,D9,D12,D13 = CONSTANTS

DIST = same as LENGTH

DKO(SO) = derivative of KO(SO) with respect to SO

DKW(SO) = derivative of KW(SO) with respect to SO

DT = Δt = incremental time step size

DX = Δx = incremental space step size

EXP = exponential

GAS = G_w = mass injection rate of water

GP = number of grid elements

KE(SO) = k_e = function definition for effective thermal conductivity

KESTR = k_e^* = effective thermal conductivity at the initial oil saturation

KO(SO) = k_o = function definition for effective permeability to oil

KW(SO) = k_w = function definition for effective permeability to water

LENGTH = cumulative space step size

MAXL = "dummy" length parameter to indicate reverse sweep ($>L-\Delta x/2$)

MUO(T) = μ_o = function definition for oil viscosity

MUOSTR = μ_o^* = viscosity of oil at initial reservoir temperature

MUW(T) = μ_w = function definition for water viscosity

MUWSTR = μ_w^* = viscosity of water at initial reservoir temperature

NABLA = ν = sensitivity of oil viscosity to temperature changes

OACCUM = oil accumulation

OIN = mass of oil injected = $G_o \Delta t$

OIPI = oil in place initially

OMB = instantaneous oil material balance

OOUT = cumulative oil production (oil out)

OOUTA = oil produced during the nth time interval

OOUTB = oil produced during the (n+1) time interval

PERM = K = absolute permeability

PHI = ϕ = hydrodynamic equation for the oil phase (function)

PHIP = ϕ_{P_K} = derivative of PHI with respect to the iteration pressure PK

PHISO = $\phi_{S_{OK}}$ = derivative of PHI with respect to the iteration saturation SOK

PK = P_K = iteration pressure

PO = P_i = initial pressure distribution

POR = ϕ = porosity

PSTR = P^* = reference pressure

PZI = ψ = hydrodynamic equation for the water phase (function)

PZIP = ψ_{P_K} = derivative of PZI with respect to the iteration pressure PK

PZISO = $\psi_{S_{OK}}$ = derivative of PZI with respect to the iteration saturation SOK

ROO(P) = ρ_o = function definition for oil density

ROOSTR = ρ_o^* = density of oil at the reference pressure (PSTR = P*)

ROROCK = ρ_r = reservoir rock density

ROW(P) = ρ_w = function definition for water density

ROWSTR = ρ_w^* = density of water at the reference pressure (PSTR = P*)

SOK = S_{OK} = iteration saturation

SOO = S_{oi} = initial oil saturation

SOXO = oil saturation at the inlet boundary (x=0)

SWC = S_{wc} = connate water saturation

SWO = S_{wi} = initial water saturation

TIME = real time

TK = T_K = iteration temperature

TLNGTH = L = total length of model

TO = T_i = initial temperature distribution

TXO = T_{INJ} = injection temperature at the inlet boundary ($x=0$)

WACCUM = water accumulation

WIN = mass of water injected = $G_w \Delta t$

WIPI = water in place initially

WMB = instantaneous water material balance

WOUT = cumulative water production (water out)

WOUTA = water produced during the nth time interval

WOUTB = water produced during the $(n+1)$ time interval

Y_1 = error in the guess of the iteration pressure PK

Y_2 = error in the guess of the iteration saturation SOK

YMAX = maximum error allowed between two successive iterations in pressure before terminating iterations

YMAX1 = maximum error allowed between two successive iterations in saturation before terminating iterations

SAMPLE COMPUTER INPUT - DATA CARDS

(1) TO,SOO,PO,GAS,SOXO,TXO

(2) POR,ROROCK,CPROCK,PERM,SWO

(3) DT,DX,GP,D8,D9,CPOSTR

(4) D12,D13,KESTR,ROOSTR,MUOSTR,ROWSTR

(5) MAXL,TLNGTH,PSTR,SWC,CPWSTR,MUWSTR

(6) NABLA

(7) CO,CW,CC1

(8) NIN

(1)	520.000	0.750	14.700	0.218	0.000	660.000
(2)	0.240	165.200	0.230	1000.000	1.000	
(3)	0.010	0.150	150.000	1.000	0.000	0.650
(4)	1.000	1.000	0.500	55.000	265.000	62.400
(5)	2.995	3.000	14.700	0.200	1.000	0.800
(6)	0.560					
(7)	$3.00\text{x}10^{-5}$	$3.20\text{x}10^{-6}$	$2.637\text{x}10^{-4}$			
(8)	10					

DATA CHANGES: NABLA = 0.560, 1.120, 1.500

DX = 0.150, 0.200, 0.300

DT = 0.010, 0.015, 0.020, 0.030

```
$IBFTC THACH   DECK,NOLIST
C       ***MAIN COMPUTER PROGRAM--THE PROGRAM WILL COMPUTE THE PRESSURE
C       AND SATURATION DISTRIBUTION IN A LINEAR,ONE-DIMENSIONAL,
C       HOMOGENEOUS RESERVOIR MODEL.  THIS INVOLVES THE SIMULTANEOUS
C       SOLUTION OF TWO NON-LINEAR PARTIAL DIFFERENTIAL EQUATIONS WITH
C       VARIABLE COEFFICIENTS.  THE TWO CONTINUITY-DARCY FLOW EQUATIONS
C       DESCRIBE THE SIMULTANEOUS FLOW OF TWO LIQUIDS, OIL AND WATER.
C       THE ALTERNATING DIRECTION EXPLICIT PROCEDURE (WRITTEN FOR ONE
C       DIMENSION) AND THE NEWTON-RAPHSON ITERATIVE TECHNIQUE WERE
C       EMPLOYED TO SOLVE THE PROBLEM.  ***
        REAL MUO,MUW,KO,KW,MUOSTR,NABLA,MUWSTR,KROT,LENGTH,MAXL,KE,KESTR
  100   FORMAT (6F12.3)
  101   FORMAT (4E15.3)
  102   FORMAT (3I8)
  104   FORMAT(1H ,28H MAXIMUM ITERATIONS EXCEEDED)
  105   FORMAT(1HK,65H DISTANCE       PRESSURE           WATER          TEMPERAT
       1URE       TIME/
       21H ,65H   FEET             PSIA        SATURATION            R
       3  HRS./
       41H ,65H --------      ---------     ----------       -----------
       5  ----/)
  106   FORMAT(1H ,F8.4,F14.3,F15.6,F16.3,F12.2)
  107   FORMAT(1HJ,19H OIL PRODUCTION IS ,F12.4,8H  POUNDS)
  108   FORMAT(1H ,21H WATER PRODUCTION IS ,F10.4,8H  POUNDS)
  109   FORMAT(1H ,40H CUMULATIVE MATERIAL BALANCE FOR OIL IS ,F13.4)
  110   FORMAT(1H ,42H CUMULATIVE MATERIAL BALANCE FOR WATER IS ,F11.4)
  201   FORMAT(1H ,43H INSTANTANEOUS MATERIAL BALANCE FOR OIL IS ,F10.4)
  202   FORMAT(1H ,45H INSTANTANEOUS MATERIAL BALANCE FOR WATER IS ,F8.4)
        COMMON TT(150),SS(150),PP(150),WFNP1,TIME,JLNGTH,DX,DT,POR
        COMMON ROOSTR,CO,PSTR,ROWSTR,CW,MUOSTR,MUWSTR,SWC,PERM,OOUT,WOUT
        COMMON CUMMBO,CUMMBW,SOO,TLNGTH,PO,AJ1(150),AJ3(150),A(55,55)
        COMMON AA(55,55),B(55),BB(55),AL(55,55),AM(55,55),AU(55,55)
        COMMON AV(55,55),DIF(55),PCENT(55),Y(55),NABLA,SW(150),CC1
        COMMON OMB,WMB,OOUTA,WOUTA,WINOUT,OOUTMB
C       ***DATA READING***
        READ (5,100) TO,SOO,PO,GAS,SOXC,TXO
        READ (5,100) POR,ROROCK,CPROCK,PERM,SWO
        READ (5,100) DT,DX,GP,D8,D9,CPOSTR
        READ (5,100) D12,D13,KESTR,ROOSTR,MUOSTR,ROWSTR
        READ (5,100) MAXL,TLNGTH,PSTR,SWC,CPWSTR,MUWSTR
        READ (5,100) NABLA
        READ (5,101) CO,CW,CC1
        READ (5,102) NIN
C       ***CONSTANTS***
        T2LONG=TLNGTH-DX/2.
        XLNGTH=TLNGTH/DX
        ILNGTH=IFIX(XLNGTH)
        IF (ABS(XLNGTH-FLOAT(ILNGTH)).GE.0.9) ILNGTH=ILNGTH+1
        JLNGTH=ILNGTH+1
        JT=JLNGTH+1
        OOUT=0.0
        WOUT=0.0
        OOUTA=0.0
        WOUTA=0.0
        WINOUT=0.0
```

```
      COUTMB=0.0
      CUMMBW=0.0
      CUMMBO=0.0
      DXDT=(POR*DX**2)/(CC1*DT)
      DDT=0.0
      XL=-0.5
C     ***PHYSICAL PROPERTY EQUATIONS***
      ROO(P)=ROOSTR*EXP(CO*(P-PSTR))
      ROW(P)=ROWSTR*EXP(CW*(P-PSTR))
      MUO(T)=MUOSTR*60.**NABLA*(T-460.)**(-NABLA)
      MUW(T)=(1776.-(T-460.))/(26.5*(T-460.)-89.)
      CPW(T)=CPWSTR
      CPO(T)=CPOSTR*(D8+D9*(T-460.))
      KE(SO)=KESTR*(D12+D13*SO)
      KO(SO)=PERM*(1.-(1.-SO-SWC)/(1.-SWC))**2*(1.-((1.-SO-SWC)/(1.-
     1 SWC))**2)
      KW(SO)=PERM*((1.-SO-SWC)/(1.-SWC))**4
      DKO(SO)=2.*PERM*((1.-(1.-SO-SWC)/(1.-SWC))**2*(1.-SO-SWC)/(1.-
     1 SWC)**2+(1.-((1.-SO-SWC)/(1.-SWC))**2)*SO/(1.-SWC)**2)
      DKW(SO)=4.*PERM*((1.-SO-SWC)/(1.-SWC))**3*(-1./(1.-SWC))
C     ***FORWARD SWEEP***
C     ***INITIAL CONDITIONS AND B.C. AT X=0***
      NGP=IFIX(GP)
      DO 1 I=1,NGP
      TT(I)=TO
      PP(I)=PO
    1 SS(I)=SOO
      SS(1)=SOXO
      TT(1)=TXO
      SW(1)=SWC
      WFNP1=GAS
C     ***BEGIN FORWARD SWEEP***
      J=1
   20 ICODE=0
      J=J+1
      DDT=DDT+1.0
      TIME=DT*DDT
      SONP1=SOXO
      TNP1=TXO
    4 PJN=PP(J)
      SOJN=SS(J)
      TJN=TT(J)
      PJP1N=PP(J+1)
      SOJP1N=SS(J+1)
      TJP1N=TT(J+1)
      KROT=KO(SONP1)
      IF (SONP1.LE.SOXO) KROT=0.0
      IF (ICODE.EQ.1) GOTO111
      XL=XL+1.0
      LENGTH=DX*XL
      IF (LENGTH.GE.MAXL) XL=XL-1.0
      IF (LENGTH.GE.MAXL) J=J-1
      IF (LENGTH.GE.MAXL) GOTO9
      GOTO112
  111 XL=XL-1.0
```

```
      LENGTH=DX*XL
 112  AJN=KO(SOJN)/MUO(TJN)
      AJP1N=KO(SOJP1N)/MUO(TJP1N)
      BJN=KW(SOJN)/MUW(TJN)
      BJP1N=KW(SOJP1N)/MUW(TJP1N)
      ANP1=KO(SONP1)/MUO(TNP1)
      BNP1=KW(SONP1)/MUW(TNP1)
C     ***ITERATION FOR PK AND SOK***
      K=0
      TK=TJN
      SOK=SS(J)-0.971*DT
      PK=PJN*1.2
  3   K=K+1
      IF (K.GT.20) GOTO5
      U1=ROO(PK)
      U2=KO(SOK)
      IF (SOK.LE.SOXO) U2=0.0
      IF (SOK.GE.0.8) U2=PERM
      U3=MUO(TK)
      U4=ROW(PK)
      U5=KW(SOK)
      IF (SOK.GE.0.8) U5=0.0
      IF (SOK.LE.SOXO) U5=PERM
      U6=MUW(TK)
      DU2SOK=DKO(SOK)
      IF (SOK.LE.SOXO) DU2SOK=0.0
      IF (SOK.GE.0.8) DU2SOK=0.0
      DU5SOK=DKW(SOK)
      IF (SOK.GE.0.8) DU5SOK=0.0
      IF (SOK.LE.SOXO) SOK=SOXO
      IF (J.GE.2) R1=1.0
      IF (J.GE.2) R2=1.0
      IF (LENGTH.GE.T2LONG) R1=4./3.
      IF (LENGTH.GE.T2LONG) R2=8./3.
      IF (J.EQ.2.AND.ICODE.EQ.1) GOTO19
      IF (J.EQ.2.AND.ICODE.EQ.0) GOTO18
      IF (ICODE.EQ.1) GOTO16
      IF (J.GT.2) DELTAP=(PNP1-PK)
C     ***NEWTON-RAPHSON ITERATIVE TECHNIQUE FOR INTERIOR GRID POINTS
C     (J GREATER THAN 2) AND FOR THE OUTLET BOUNDARY ON THE FORWARD
C     SWEEP.  ***
      PHI=R1*(U2/U3+ANP1)*DELTAP/2.+R2*(AJN+AJP1N)*(PJP1N-PJN)/2.-DXDT*
     1 (SOK-SOJN+CO*(SOK+SOJN)*(PK-PJN)/2.)
      PZI=R1*(U5/U6+BNP1)*DELTAP/2.+R2*(BJN+BJP1N)*(PJP1N-PJN)/2.-DXDT*
     1 (SOJN-SOK+CW*(1.-(SOK+SOJN)/2.)*(PK-PJN))
C     ***ELEMENTS OF ITERATION MATRIX***
      PHIP=R1*(U2/U3+ANP1)/(-2.)-DXDT*(CO*(SOK+SOJN)/2.)
      PZIP=R1*(U5/U6+BNP1)/(-2.)-DXDT*(CW*(1.-(SOK+SOJN)/2.))
 31   PHISO=R1*DU2SOK*DELTAP/(2.*U3)-DXDT*(1.+CO*(PK-PJN)/2.)
      PZISO=R1*DU5SOK*DELTAP/(2.*U6)+DXDT*(1.+CW*(PK-PJN)/2.)
      GOTO90
C     ***NEWTON-RAPHSON ITERATIVE TECHNIQUE FOR THE INLET BOUNDARY
C     (J EQUAL TO 2) ON THE FORWARD SWEEP.  ***
 18   PHI=(AJN+AJP1N)*(PJP1N-PJN)/2.-DXDT*(SOK-SOJN+CO*(SOK+SOJN)*
     1 (PK-PJN)/2.)
```

```
      PZI=WFNP1*DX/(CC1*ROW(PK))+(BJN+BJP1N)*(PJP1N-PJN)/2.-DXDT*
     1 (SOJN-SOK+CW*(1.-(SOK+SOJN)/2.)*(PK-PJN))
      PHIP=-DXDT*(CO*(SOK+SOJN)/2.)
      PZIP=-DXDT*(CW*(1.-(SOK+SOJN)/2.))
      PHISO=-DXDT*(1.+CO*(PK-PJN)/2.)
      PZISO=DXDT*(1.+CW*(PK-PJN)/2.)
      GOTO90
C     ***NEWTON-RAPHSON ITERATIVE TECHNIQUE FOR THE INLET BOUNDARY
C     (J EQUAL TO 2) ON THE REVERSE SWEEP.  ***
 19   PHI=(U2/U3+AJP1N)*(PJP1N-PK)/2.-DXDT*(SOK-SOJN+CO*(SOK+SOJN)*
     1 (PK-PJN)/2.)
      PZI=WFNP1*DX/(CC1*ROW(PJN))+(U5/U6+BJP1N)*(PJP1N-PK)/2.-DXDT*
     1 (SOJN-SOK+CW*(1.-(SOK+SOJN)/2.)*(PK-PJN))
      GOTO98
C     ***NEWTON-RAPHSON ITERATIVE TECHNIQUE FOR INTERIOR GRID POINTS
C     (J GREATER THAN 2) AND FOR THE OUTLET BOUNDARY ON THE REVERSE
C     SWEEP.  ***
 16   IF (J.GT.2) DELTAP=(PNP1-PJN)
      PHI=R1*(AJN+ANP1)*DELTAP/2.+R2*(U2/U3+AJP1N)*(PJP1N-PK)/2.-DXDT*
     1 (SOK-SOJN+CO*(SOK+SOJN)*(PK-PJN)/2.)
      PZI=R1*(BJN+BNP1)*DELTAP/2.+R2*(U5/U6+BJP1N)*(PJP1N-PK)/2.-DXDT*
     1 (SOJN-SOK+CW*(1.-(SOK+SOJN)/2.)*(PK-PJN))
C     ***ELEMENTS OF ITERATION MATRIX***
 98   PHIP=R2*(U2/U3+AJP1N)/(-2.)-DXDT*(CO*(SOK+SOJN)/2.)
      PZIP=R2*(U5/U6+BJP1N)/(-2.)-DXDT*(CW*(1.-(SOK+SOJN)/2.))
      PHISO=R2*DU2SOK*(PJP1N-PK)/(2.*U3)-DXDT*(1.+CO*(PK-PJN)/2.)
      PZISO=R2*DU5SOK*(PJP1N-PK)/(2.*U6)+DXDT*(1.+CW*(PK-PJN)/2.)
 90   Y1=(PZI*PHISO-PHI*PZISO)/(PZISO*PHIP-PZIP*PHISO)
      Y2=(PZI*PHIP-PHI*PZIP)/(PZIP*PHISO-PHIP*PZISO)
      YMAX=1.0E-04
      YMAX1=1.0E-06
      IF (ABS(Y1/PK).LE.YMAX.AND.ABS(Y2/SOK).LE.YMAX1) GOTO2
      PK=PK+Y1
      SOK=SOK+Y2
      GOTO3
  2   CONTINUE
      IF (J.GT.2) GOTO187
      PP(1)=DELTAP+PK
      PB=PP(1)
 187  PP(J)=PK
      SS(J)=SOK
      SWK=1.-SOK
      SW(J)=SWK
      TT(J)=TK
      ROCP=(U4*CPW(TK)*(1.-SOK)+U1*CPO(TK)*SOK)*POR+ROROCK*CPROCK*(1.-
     1 POR)
      ACV=(U1*CPO(TK)*U2/U3+U4*CPW(TK)*U5/U6)
      AJ1(J)=2.*ROCP*DX**2/(CC1*ACV*DT)
      AJ3(J)=KE(SOK)/(CC1*ACV)
 181  IF (ICODE.EQ.1) J=J-1
      JS=J
      IF (J.EQ.1.AND.ICODE.EQ.1) XL=XL-1.0
      IF (J.EQ.1.AND.ICODE.EQ.1) GOTO185
      IF (ICODE.EQ.1) GOTO93
      PNP1=PP(J)
```

```
      SONP1=SS(J)
      TNP1=TT(J)
      J=J+1
      GOTO4
 185  CALL MATBAL
      CALL TEMP
      CALL SOLUT
      NDDT=IFIX(DDT)
      NDDT2=NDDT/NIN
      IF (NDDT2.EQ.1) GOTO180
      J=JS
      GOTO20
 180  WRITE (6,105)
      DIST=0.0000
      DO 400 JW=1,JT
      WRITE (6,106) DIST,PP(JW),SW(JW),TT(JW),TIME
      XX=DX
      IF (JW.EQ.1.OR.JW.EQ.(JT-1)) XX=DX/2.
      DIST=DIST+XX
 400  CONTINUE
      WRITE (6,107) OOUT
      WRITE (6,108) WOUT
      WRITE (6,109) CUMMBO
      WRITE (6,110) CUMMBW
      WRITE (6,201) OMB
      WRITE (6,202) WMB
      NIN=NIN+10
      J=JS
      GOTO20
C     ***REVERSE SWEEP***
 9    ICODE=1
      DDT=DDT+1.0
      TIME=DT*DDT
      XL=XL+1.0
      KIK=J+1
      SS(KIK)=SS(J)
      SW(KIK)=1.-SS(KIK)
 93   PNP1=PP(J-1)
      SONP1=SS(J-1)
      TNP1=TT(J-1)
      GOTO4
 5    WRITE (6,104)
      CALL EXIT
      END
```

```
$IBFTC MAY    DECK,NOLIST
      SUBROUTINE MATBAL
C     ***MATERIAL BALANCE SUBROUTINE--A CUMULATIVE AND INSTANTANEOUS
C     MATERIAL BALANCE IS CARRIED OUT FOR EACH PHASE (OIL AND WATER)
C     AS A CHECK ON THE MAIN PROGRAM RESULTS.  DEVIATIONS FROM 1.0 WILL
C     INDICATE THE FRACTIONAL ERROR IN THE MATERIAL BALANCE.  ***
      REAL MUO,MUW,KO,KW,MUOSTR,NABLA,MUWSTR,KROT,LENGTH,MAXL,KE,KESTR
      COMMON TT(150),SS(150),PP(150),WFNP1,TIME,JLNGTH,DX,DT,POR
      COMMON ROOSTR,CO,PSTR,ROWSTR,CW,MUOSTR,MUWSTR,SWC,PERM,OOUT,WOUT
      COMMON CUMMBO,CUMMBW,SOO,TLNGTH,PO,AJ1(150),AJ3(150),A(55,55)
      COMMON AA(55,55),B(55),BB(55),AL(55,55),AM(55,55),AU(55,55)
      COMMON AV(55,55),DIF(55),PCENT(55),Y(55),NABLA,SW(150),CC1
      COMMON OMB,WMB,OOUTA,WOUTA,WINOUT,OOUTMB
  101 FORMAT (1H ,8E15.5)
      ROO(P)=ROOSTR*EXP(CO*(P-PSTR))
      ROW(P)=ROWSTR*EXP(CW*(P-PSTR))
      MUO(T)=MUOSTR*60.**NABLA*(T-460.)**(-NABLA)
      MUW(T)=(1776.-(T-460.))/(26.5*(T-460.)-89.)
      KO(SO)=PERM*(1.-(1.-SO-SWC)/(1.-SWC))**2*(1.-((1.-SO-SWC)/(1.-
     1 SWC))**2)
      KW(SO)=PERM*((1.-SO-SWC)/(1.-SWC))**4
      OIN=0.0
      OIPI=TLNGTH*POR*ROO(PO)*SOO
      WIN=WFNP1*TIME
      WIPI=TLNGTH*POR*ROW(PO)*(1.-SOO)
      WACCUM=0.0
      OACCUM=0.0
      DO 190 I=2,JLNGTH
      TL=TT(I)
      SL=SS(I)
      PL=PP(I)
      OACCUM=OACCUM+DX*POR*SL*ROO(PL)
  190 WACCUM=WACCUM+DX*POR*(1.-SL)*ROW(PL)
      PLM1=PP(JLNGTH)
      PL=PO
      PR=(PLM1+PL)/2.
      WOUTB=(ROW(PR)*KW(SL)/MUW(TL))*(PLM1-PL)*2.*DT*CC1/(DX/2.)
      OOUTB=(ROO(PR)*KO(SL)/MUO(TL))*(PLM1-PL)*2.*DT*CC1/(DX/2.)
      WOUTAV=(WOUTA+WOUTB)/2.
      OOUTAV=(OOUTA+OOUTB)/2.
      IF (TIME.EQ.2.*DT) WOUTAV=WOUTB
      IF (TIME.EQ.2.*DT) OOUTAV=OOUTB
      IF (TIME.EQ.2.*DT) HACCUM=WIPI
      IF (TIME.EQ.2.*DT) GACCUM=OIPI
      WMB=(WACCUM-HACCUM)/(WFNP1*2.*DT-WOUTAV)
      OMB=(GACCUM-OACCUM)/OOUTAV
      HACCUM=WACCUM
      GACCUM=OACCUM
      WOUT=WOUT+WOUTAV
      OOUT=OOUT+OOUTAV
      OOUTMB=OIPI-OACCUM
      WINOUT=WACCUM-WIPI
      CUMMBW=WINOUT/(WIN-WOUT)
      WOUTA=WOUTB
      OOUTA=OOUTB
```

```
      IF (OOUT.LE.0.0) GOTO186
      CUMMBC=OOUTMB/OOUT
  186 RETURN
      END
```

```
$IBFTC MAYBE   DECK,NOLIST
      SUBROUTINE TEMP
C     ***MATRIX FORMULATION FOR TEMPERATURE DISTRIBUTION-- THIS
C     SUBROUTINE USES THE SATURATION AND PRESSURE DISTRIBUTIONS
C     OBTAINED FROM THE MAIN PROGRAM TO FORM A MATRIX OF THERMAL
C     ENERGY EQUATIONS (CRANK-NICOLSON FINITE DIFFERENCE EQUATIONS)
C     WHICH WHEN SOLVED WILL YIELD A NEW TEMPERATURE DISTRIBUTION.  ***
      REAL MUO,MUW,KO,KW,MUOSTR,NABLA,MUWSTR,KROI,LENGTH,MAXL,KE,KESTR
      COMMON TT(150),SS(150),PP(150),WFNP1,TIME,JLNGTH,DX,DT,POR
      COMMON ROOSTR,CC,PSTR,ROWSTR,CW,MUOSTR,MUWSTR,SWC,PERM,OOUT,WOUT
      COMMON CUMMBO,CUMMBW,SOO,TLNGTH,PO,AJ1(150),AJ3(150),A(55,55)
      COMMON AA(55,55),B(55),BB(55),AL(55,55),AM(55,55),AU(55,55)
      COMMON AV(55,55),DIF(55),PCENT(55),Y(55),NABLA,SW(150),CC1
      COMMON OMB,WMB,OOUTA,WOUTA,WINOUT,OOUTMB
 101  FORMAT (1H ,8E15.5)
      JJLNG=JLNGTH+1
      DO 39 I=1,JJLNG
      DO 39 J=1,JJLNG
 39   A(I,J)=0.0
      MLNGTH=JLNGTH-1
      J=2
      I=1
 79   AJ2=-PP(J+2)+6.*PP(J+1)-3.*PP(J)-2.*PP(J-1)
      AJ4=-TT(J+2)+6.*TT(J+1)-3.*TT(J)-2.*TT(J-1)
      AJ5=TT(J+1)-2.*TT(J)+TT(J-1)
      BB1=AJ1(J)+AJ2/12.+2.*AJ3(J)
      BB2=-AJ2/6.-AJ3(J)
      BB3=AJ2/36.
      BB4=AJ2/18.-AJ3(J)
      BB5=AJ1(J)*TT(J)+BB3*AJ4+AJ3(J)*AJ5
      IF (J.EQ.2) GOTO78
      IF (J.EQ.3) GOTO77
      IF (J.EQ.4) GOTO76
      GOTO75
 161  AJ2=2.*PP(J+1)+3.*PP(J)-6.*PP(J-1)+PP(J-2)
      AJ4=2.*TT(J+1)+3.*TT(J)-6.*TT(J-1)+TT(J-2)
      AJ5=TT(J+1)-2.*TT(J)+TT(J-1)
      BB2=AJ1(J)-AJ2/12.+2.*AJ3(J)
      BB3=-AJ2/18.-AJ3(J)
      BB4=-AJ2/36.
      BB1=AJ2/6.-AJ3(J)
      BB5=AJ1(J)*TT(J)-BB3*AJ4+AJ3(J)*AJ5
      GOTO140
 78   A(1,1)=BB1
      A(1,2)=BB2
      A(1,3)=BB3
      B(I)=BB5-BB4*TT(J-1)
      J=J+1
      I=I+1
      GOTO79
 77   A(2,1)=BB4
      A(2,2)=BB1
      A(2,3)=BB2
      A(2,4)=BB3
      B(I)=BB5
```

```
      J=J+1
      I=I+1
      GOTO79
76    L=3
      MMM=0
      M=1
44    NN=1
49    M=M+NN
      MM=M-MMM
      IF (MM.EQ.2) GOTO43
      IF (MM.EQ.3) GOTO45
      IF (MM.EQ.4) GOTO46
      IF (MM.EQ.5) GOTO47
      GOTO48
74    B(I)=BB5
      GOTO173
43    A(L,M)=BB4
      GOTO44
45    A(L,M)=BB1
      GOTO44
46    A(L,M)=BB2
      IF (J.GE.MLNGTH) A(L,M)=BB2+BB3
      IF (J.EQ.MLNGTH) B(I)=BB5
      IF (J.EQ.MLNGTH) GOTO48
      IF (J.EQ.JLNGTH) GOTO74
      GOTO44
47    A(L,M)=BB3
      B(I)=BB5
      GOTO44
48    J=J+1
      I=I+1
      IF (J.EQ.JLNGTH) GOTO161
      GOTO79
75    L=L+1
      NN=L-2
      M=1
      MMM=MMM+1
      GOTO49
140   L=L+1
      NN=L-3
      M=1
      GOTO49
173   RETURN
      END
```

```
$IBFTC LOLA    DECK,NOLIST
      SUBROUTINE SOLUT
C     ***INVERSION OF A NON-SYMMETRIC MATRIX BY A MODIFIED SQUARE ROOT
C     METHOD--THIS SUBROUTINE INVERTS THE MATRIX SET UP BY SUBROUTINE
C     TEMP AND THEN SOLVES FOR THE NEW TEMPERATURE DISTRIBUTION
C     WHICH IS THEN USED IN THE MAIN PROGRAM TO CALCULATE THE TEMPERATURE
C     DEPENDENT PROPERTIES FOR THE NEXT TIME INTERVAL.  ***
      REAL MUO,MUW,KO,KW,MUOSTR,NABLA,MUWSTR,KROT,LENGTH,MAXL,KE,KESTR
      COMMON TT(150),SS(150),PP(150),WFNP1,TIME,JLNGTH,DX,DT,POR
      COMMON ROOSTR,CO,PSTR,ROWSTR,CW,MUOSTR,MUWSTR,SWC,PERM,OOUT,WOUT
      COMMON CUMMBO,CUMMBW,SOO,TLNGTH,PO,AJ1(150),AJ3(150),A(55,55)
      COMMON AA(55,55),B(55),BB(55),AL(55,55),AM(55,55),AU(55,55)
      COMMON AV(55,55),DIF(55),PCENT(55),Y(55),NABLA,SW(150),CC1
      COMMON OMB,WMB,OOUTA,WOUTA,WINCUT,OOUTMB
 100  FORMAT (1H ,8E15.5)
  201 FORMAT (14X,14HSOLUTION ERROR/14X,6F15.6)
      CALL FPTSW
      M=JLNGTH-1
      DO 1 I=1,M
      DO 2 J=1,M
 2    AA(I,J)=A(I,J)
 1    BB(I)=B(I)
C     CALCULATION OF L AND U
      DO 105 K=2,M
      KK=K-1
      DO 104 J=1,KK
      AL(J,K)=0.0
      AM(J,K)=0.0
      AU(K,J)=0.0
  104 AV(K,J)=0.0
  105 CONTINUE
      IF(A(1,1).EQ.0.) GO TO 150
  106 XIS=ABS(A(1,1))
      AL(1,1)=SQRT(XIS)
      AU(1,1)=A(1,1)/AL(1,1)
      DO 107 J=2,M
      AU(1,J)=A(1,J)/AL(1,1)
  107 AL(J,1)=A(J,1)/AU(1,1)
      DO 115 K=2,M
      KK=K-1
      VALUE=0.0
      DO 108 J=1,KK
  108 VALUE=VALUE+AL(K,J)*AU(J,K)
      ZX=A(K,K)-VALUE
      IF(ZX.EQ.0.) GO TO 150
  109 ZXX=ABS(ZX)
      AL(K,K)=SQRT(ZXX)
      AU(K,K)=ZX/AL(K,K)
      KP=K+1
      KK=K-1
      IF(KP.GT.M ) GO TO 115
      DO 112 I=KP,M
      ZV=0.0
      ZW=0.0
      DO 110 LP=1,KK
```

```
      ZV=ZV+AL(K,LP)*AU(LP,I)
  110 ZW=ZW+AL(I,LP)*AU(LP,K)
      AU(K,I)=(A(K,I)-ZV)/AL(K,K)
  112 AL(I,K)=(A(I,K)-ZW)/AU(K,K)
  115 CONTINUE
C     L AND U ARE CALCULATED
C     PROCEEDING TO CALCULATE L-INVERSE AND U-INVERSE
      DO 119 K=1,M
      AM(K,K)=1.0/AL(K,K)
  119 AV(K,K)=1.0/AU(K,K)
      DO 125 K=2,M
      KK=K-1
      DO 122 J=1,KK
      ZQ=0.0
      DO 120 L=J,KK
  120 ZQ=ZQ+AL(K,L)*AM(L,J)
  122 AM(K,J)=-ZQ/AL(K,K)
  125 CONTINUE
      IJN=M+1
      DO 135 KL=1,IJN
      K=IJN-KL
      IF(K.LE.1) GO TO 136
  126 DO 130 JK=1,K
      J=K-JK
      IF(J.LT.1) GO TO 135
  127 ZR=0.0
      JP=J+1
      DO 128 L=JP,K
  128 ZR=ZR+AU(J,L)*AV(L,K)
  130 AV(J,K)=-ZR/AU(J,J)
  135 CONTINUE
C     PROCEEDING TO CALCULATE A-INVERSE=U-INVERSE*L-INVERSE
  136 DO 140 K=1,M
      DO 140 J=1,M
      A(J,K)=0.0
      DO 137 L=1,M
      A(J,K)=A(J,K)+AV(J,L)*AM(L,K)
 137  CONTINUE
  140 CONTINUE
      GO TO 152
  150 WRITE(6,151)
  151 FORMAT(1H ,29H THE METHOD IS NOT APPLICABLE)
  152 CONTINUE
      DO 153 I=1,M
      Y(I)=0.
      DO 153 J=1,M
 153  Y(I)=Y(I)+A(I,J)*B(J)
      DO 154 I=1,JLNGTH
      KK=I+1
 154  TT(KK)=Y(I)
      JIJ=JLNGTH+1
      TT(JIJ)=TT(JLNGTH)
      CALL FPTSW
      DO 9 J=1,M
      BOB=0.
```

```
      DO 12 I=1,M
   12 BOB=BOB+AA(J,I)*Y(I)
      DIF(J)=BB(J)-BOB
      PCENT(J)=DIF(J)*100./BB(J)
    9 CONTINUE
  202 RETURN
      END
```

COMPUTER OUTPUT I

√ = NABLA = 0.560

Δx = DX = 0.300 feet

Δt = DT = 0.015 Hrs.

```
DISTANCE       PRESSURE         WATER          TEMPERATURE       TIME
  FEET           PSIA         SATURATION            R            HRS.
--------      ---------       ----------       -----------       ----

 0.0000         25.448         1.000000          520.000         9.00
 0.1500         25.066         0.482922          520.000         9.00
 0.4500         24.686         0.407252          520.000         9.00
 0.7500         23.791         0.294014          520.000         9.00
 1.0500         22.537         0.251132          520.000         9.00
 1.3500         21.330         0.250006          520.000         9.00
 1.6500         20.124         0.249998          520.000         9.00
 1.9500         18.919         0.249999          520.000         9.00
 2.2500         17.713         0.250000          520.000         9.00
 2.5500         16.508         0.250001          520.000         9.00
 2.8500         15.303         0.250001          520.000         9.00
 3.0000         14.700         0.250001          520.000         9.00

OIL PRODUCTION IS        1.6944  POUNDS
WATER PRODUCTION IS      0.0078  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS          1.0146
CUMULATIVE MATERIAL BALANCE FOR WATER IS        1.0010
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS       0.9995
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS     1.0051
```

```
DISTANCE       PRESSURE         WATER          TEMPERATURE       TIME
  FEET           PSIA         SATURATION            R            HRS.
--------      ---------       ----------       -----------       ----

 0.0000         24.415         1.000000          660.000         9.00
 0.1500         24.193         0.464256          589.494         9.00
 0.4500         23.964         0.407581          545.958         9.00
 0.7500         23.297         0.303895          527.804         9.00
 1.0500         22.148         0.251841          521.927         9.00
 1.3500         21.009         0.250015          520.391         9.00
 1.6500         19.864         0.249999          520.061         9.00
 1.9500         18.716         0.249999          520.001         9.00
 2.2500         17.569         0.250000          519.992         9.00
 2.5500         16.421         0.250001          519.994         9.00
 2.8500         15.274         0.250002          519.999         9.00
 3.0000         14.700         0.250002          519.999         9.00

OIL PRODUCTION IS        1.6328  POUNDS
WATER PRODUCTION IS      0.0075  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS          1.0343
CUMULATIVE MATERIAL BALANCE FOR WATER IS        0.9830
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS       1.0112
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS     0.9682
```

```
DISTANCE       PRESSURE          WATER          TEMPERATURE        TIME
  FEET           PSIA          SATURATION            R             HRS.
--------      ---------        ----------       -----------        ----

 0.0000         21.992          1.000000          520.000          18.00
 0.1500         21.815          0.525016          520.000          18.00
 0.4500         21.608          0.474276          520.000          18.00
 0.7500         21.299          0.437938          520.000          18.00
 1.0500         20.795          0.355678          520.000          18.00
 1.3500         19.881          0.262356          520.000          18.00
 1.6500         18.928          0.250148          520.000          18.00
 1.9500         17.989          0.250003          520.000          18.00
 2.2500         17.049          0.250002          520.000          18.00
 2.5500         16.110          0.250003          520.000          18.00
 2.8500         15.170          0.250004          520.000          18.00
 3.0000         14.700          0.250004          520.000          18.00

OIL PRODUCTION IS          3.1353  POUNDS
WATER PRODUCTION IS        0.0144  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS              1.0163
CUMULATIVE MATERIAL BALANCE FOR WATER IS            0.9256
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS           1.0222
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS         0.8016
```

```
DISTANCE       PRESSURE          WATER          TEMPERATURE        TIME
  FEET           PSIA          SATURATION            R             HRS.
--------      ---------        ----------       -----------        ----

 0.0000         22.132          1.000000          660.000          18.00
 0.1500         22.042          0.500967          609.092          18.00
 0.4500         21.930          0.463910          567.487          18.00
 0.7500         21.741          0.442757          541.779          18.00
 1.0500         21.361          0.389864          528.778          18.00
 1.3500         20.535          0.284928          523.159          18.00
 1.6500         19.458          0.250794          520.999          18.00
 1.9500         18.404          0.250011          520.271          18.00
 2.2500         17.347          0.250003          520.058          18.00
 2.5500         16.288          0.250003          520.009          18.00
 2.8500         15.230          0.250004          520.012          18.00
 3.0000         14.700          0.250004          520.012          18.00

OIL PRODUCTION IS          3.2311  POUNDS
WATER PRODUCTION IS        0.0148  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS              1.0201
CUMULATIVE MATERIAL BALANCE FOR WATER IS            0.9577
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS           1.0048
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS         0.8880
```

```
DISTANCE        PRESSURE          WATER           TEMPERATURE       TIME
  FEET            PSIA          SATURATION             R            HRS.
--------        ---------       ----------       -----------        ----

 0.0000          20.791          1.000000           520.000         27.00
 0.1500          20.663          0.548802           520.000         27.00
 0.4500          20.504          0.494805           520.000         27.00
 0.7500          20.291          0.469797           520.000         27.00
 1.0500          20.019          0.450668           520.000         27.00
 1.3500          19.606          0.397898           520.000         27.00
 1.6500          18.869          0.291054           520.000         27.00
 1.9500          17.915          0.251095           520.000         27.00
 2.2500          16.995          0.250014           520.000         27.00
 2.5500          16.077          0.250005           520.000         27.00
 2.8500          15.159          0.250006           520.000         27.00
 3.0000          14.700          0.250006           520.000         27.00

OIL PRODUCTION IS         4.4804  POUNDS
WATER PRODUCTION IS       0.0205  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS             1.0195
CUMULATIVE MATERIAL BALANCE FOR WATER IS           0.8841
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS          1.0258
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS        0.7855


DISTANCE        PRESSURE          WATER           TEMPERATURE       TIME
  FEET            PSIA          SATURATION             R            HRS.
--------        ---------       ----------       -----------        ----

 0.0000          22.500          1.000000           660.000         27.00
 0.1500          22.384          0.521968           618.755         27.00
 0.4500          22.306          0.484723           581.751         27.00
 0.7500          22.080          0.467383           554.304         27.00
 1.0500          21.779          0.455531           537.079         27.00
 1.3500          21.368          0.435051           527.752         27.00
 1.6500          20.703          0.364465           523.242         27.00
 1.9500          19.458          0.266967           521.249         27.00
 2.2500          18.084          0.250245           520.434         27.00
 2.5500          16.730          0.250008           520.139         27.00
 2.8500          15.374          0.250006           520.038         27.00
 3.0000          14.700          0.250006           520.038         27.00

OIL PRODUCTION IS         4.8443  POUNDS
WATER PRODUCTION IS       0.0222  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS             1.0181
CUMULATIVE MATERIAL BALANCE FOR WATER IS           0.9550
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS          1.0180
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS        1.1458
```

```
DISTANCE        PRESSURE          WATER            TEMPERATURE        TIME
  FEET            PSIA          SATURATION              R             HRS.
--------       ---------        ----------        -----------         ----

 0.0000          19.594          1.000000           520.000          36.00
 0.1500          19.490          0.566115           520.000          36.00
 0.4500          19.357          0.509379           520.000          36.00
 0.7500          19.182          0.481683           520.000          36.00
 1.0500          18.965          0.467172           520.000          36.00
 1.3500          18.714          0.456064           520.000          36.00
 1.6500          18.381          0.423168           520.000          36.00
 1.9500          17.802          0.328627           520.000          36.00
 2.2500          16.899          0.255227           520.000          36.00
 2.5500          16.016          0.250059           520.000          36.00
 2.8500          15.139          0.250008           520.000          36.00
 3.0000          14.700          0.250008           520.000          36.00

OIL PRODUCTION IS        5.7694  POUNDS
WATER PRODUCTION IS      0.0265  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS            1.0206
CUMULATIVE MATERIAL BALANCE FOR WATER IS          0.8545
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS         1.0236
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS       0.7488



DISTANCE        PRESSURE          WATER            TEMPERATURE        TIME
  FEET            PSIA          SATURATION              R             HRS.
--------       ---------        ----------        -----------         ----

 0.0000          15.994          1.000000           660.000          36.00
 0.1500          15.955          0.536567           624.920          36.00
 0.4500          15.893          0.501980           591.777          36.00
 0.7500          15.821          0.481531           564.628          36.00
 1.0500          15.759          0.469052           545.315          36.00
 1.3500          15.694          0.460466           533.193          36.00
 1.6500          15.619          0.451330           526.376          36.00
 1.9500          15.514          0.417384           522.885          36.00
 2.2500          15.332          0.318794           521.219          36.00
 2.5500          15.077          0.253643           520.416          36.00
 2.8500          14.831          0.250058           519.854          36.00
 3.0000          14.700          0.250058           519.854          36.00

OIL PRODUCTION IS        6.3702  POUNDS
WATER PRODUCTION IS      0.0292  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS            1.0199
CUMULATIVE MATERIAL BALANCE FOR WATER IS          0.9428
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS         1.1738
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS       0.2564
```

```
 DISTANCE        PRESSURE         WATER          TEMPERATURE       TIME
   FEET            PSIA        SATURATION             R            HRS.
 --------       ---------      ----------       -----------        ----

  0.0000          18.583         1.000000          520.000         45.00
  0.1500          18.494         0.579732          520.000         45.00
  0.4500          18.379         0.521341          520.000         45.00
  0.7500          18.227         0.491833          520.000         45.00
  1.0500          18.037         0.474753          520.000         45.00
  1.3500          17.817         0.465370          520.000         45.00
  1.6500          17.575         0.458321          520.000         45.00
  1.9500          17.283         0.437631          520.000         45.00
  2.2500          16.810         0.363214          520.000         45.00
  2.5500          15.999         0.265950          520.000         45.00
  2.8500          15.128         0.250295          520.000         45.00
  3.0000          14.700         0.250292          520.000         45.00

 OIL PRODUCTION IS         7.0126   POUNDS
 WATER PRODUCTION IS       0.0322   POUNDS
 CUMULATIVE MATERIAL BALANCE FOR OIL IS            1.0210
 CUMULATIVE MATERIAL BALANCE FOR WATER IS          0.8310
 INSTANTANEOUS MATERIAL BALANCE FOR OIL IS         1.0226
 INSTANTANEOUS MATERIAL BALANCE FOR WATER IS       0.7284
```

```
 DISTANCE        PRESSURE         WATER          TEMPERATURE       TIME
   FEET            PSIA        SATURATION             R            HRS.
 --------       ---------      ----------       -----------        ----

  0.0000          13.218         1.000000          660.000         45.00
  0.1500          13.216         0.548364          628.759         45.00
  0.4500          13.162         0.504980          597.783         45.00
  0.7500          13.160         0.481586          571.448         45.00
  1.0500          13.197         0.468247          551.617         45.00
  1.3500          13.265         0.459386          538.095         45.00
  1.6500          13.359         0.448716          529.678         45.00
  1.9500          13.494         0.406644          524.856         45.00
  2.2500          13.749         0.297741          522.206         45.00
  2.5500          14.135         0.251512          520.511         45.00
  2.8500          14.513         0.250035          518.668         45.00
  3.0000          14.700         0.250035          518.668         45.00

 OIL PRODUCTION IS         6.2022   POUNDS
 WATER PRODUCTION IS       0.0284   POUNDS
 CUMULATIVE MATERIAL BALANCE FOR OIL IS            1.0327
 CUMULATIVE MATERIAL BALANCE FOR WATER IS          0.7428
 INSTANTANEOUS MATERIAL BALANCE FOR OIL IS         0.8477
 INSTANTANEOUS MATERIAL BALANCE FOR WATER IS      -0.2606
```

COMPUTER OUTPUT II

√ = NABLA = 1.120

Δx = DX = 0.300 feet

Δt = DT = 0.020 Hrs.

```
DISTANCE        PRESSURE           WATER            TEMPERATURE        TIME
  FEET            PSIA          SATURATION               R             HRS.
--------        ---------       ----------          -----------        ----

 0.0000          25.341          1.000000             520.000           9.0
 0.1500          24.965          0.482909             520.000           9.0
 0.4500          24.585          0.406842             520.000           9.0
 0.7500          23.706          0.293873             520.000           9.0
 1.0500          22.463          0.251126             520.000           9.0
 1.3500          21.267          0.250005             520.000           9.0
 1.6500          20.073          0.249997             520.000           9.0
 1.9500          18.879          0.249998             520.000           9.0
 2.2500          17.685          0.249999             520.000           9.0
 2.5500          16.491          0.250000             520.000           9.0
 2.8500          15.297          0.250000             520.000           9.0
 3.0000          14.700          0.250000             520.000           9.0

OIL PRODUCTION IS          1.6768  POUNDS
WATER PRODUCTION IS        0.0077  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS              1.0239
CUMULATIVE MATERIAL BALANCE FOR WATER IS            0.9996
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS           0.9962
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS         0.9930
```

```
DISTANCE        PRESSURE           WATER            TEMPERATURE        TIME
  FEET            PSIA          SATURATION               R             HRS.
--------        ---------       ----------          -----------        ----

 0.0000          23.822          1.000000             660.000           9.0
 0.1500          23.636          0.474959             589.324           9.0
 0.4500          23.437          0.401498             545.928           9.0
 0.7500          22.805          0.291652             527.817           9.0
 1.0500          21.753          0.250992             521.930           9.0
 1.3500          20.682          0.250004             520.394           9.0
 1.6500          19.597          0.249998             520.065           9.0
 1.9500          18.509          0.249999             520.005           9.0
 2.2500          17.421          0.249999             519.996           9.0
 2.5500          16.332          0.250000             519.996           9.0
 2.8500          15.244          0.250000             519.999           9.0
 3.0000          14.700          0.250000             519.999           9.0

OIL PRODUCTION IS          1.5253  POUNDS
WATER PRODUCTION IS        0.0070  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS              1.0853
CUMULATIVE MATERIAL BALANCE FOR WATER IS            0.9633
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS           1.0307
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS         0.9353
```

```
 DISTANCE        PRESSURE          WATER          TEMPERATURE        TIME
   FEET            PSIA         SATURATION             R             HRS.
 --------       ---------       ----------        -----------        ----

  0.0000         22.889          1.000000           520.000          18.0
  0.1500         22.718          0.524720           520.000          18.0
  0.4500         22.511          0.474521           520.000          18.0
  0.7500         22.226          0.444946           520.000          18.0
  1.0500         21.730          0.379558           520.000          18.0
  1.3500         20.764          0.274996           520.000          18.0
  1.6500         19.636          0.250429           520.000          18.0
  1.9500         18.539          0.250003           520.000          18.0
  2.2500         17.443          0.250001           520.000          18.0
  2.5500         16.346          0.250001           520.000          18.0
  2.8500         15.250          0.250002           520.000          18.0
  3.0000         14.700          0.250002           520.000          18.0

 OIL PRODUCTION IS          3.3338  POUNDS
 WATER PRODUCTION IS        0.0153  POUNDS
 CUMULATIVE MATERIAL BALANCE FOR OIL IS           1.0076
 CUMULATIVE MATERIAL BALANCE FOR WATER IS         0.9761
 INSTANTANEOUS MATERIAL BALANCE FOR OIL IS        0.9872
 INSTANTANEOUS MATERIAL BALANCE FOR WATER IS      0.9052



 DISTANCE        PRESSURE          WATER          TEMPERATURE        TIME
   FEET            PSIA         SATURATION             R             HRS.
 --------       ---------       ----------        -----------        ----

  0.0000         20.860          1.000000           660.000          18.0
  0.1500         20.812          0.518796           608.885          18.0
  0.4500         20.719          0.464119           567.152          18.0
  0.7500         20.598          0.433386           541.673          18.0
  1.0500         20.255          0.359360           528.789          18.0
  1.3500         19.488          0.264420           523.174          18.0
  1.6500         18.628          0.250182           521.006          18.0
  1.9500         17.762          0.250002           520.277          18.0
  2.2500         16.890          0.250001           520.064          18.0
  2.5500         16.016          0.250002           520.012          18.0
  2.8500         15.142          0.250002           520.010          18.0
  3.0000         14.700          0.250002           520.010          18.0

 OIL PRODUCTION IS          2.9877  POUNDS
 WATER PRODUCTION IS        0.0137  POUNDS
 CUMULATIVE MATERIAL BALANCE FOR OIL IS           1.0465
 CUMULATIVE MATERIAL BALANCE FOR WATER IS         0.9080
 INSTANTANEOUS MATERIAL BALANCE FOR OIL IS        0.9886
 INSTANTANEOUS MATERIAL BALANCE FOR WATER IS      0.7277
```

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DISTANCE       PRESSURE        WATER         TEMPERATURE      TIME
  FEET           PSIA        SATURATION           R           HRS.
--------      ---------      ----------      -----------      ----

 0.0000        18.124         1.000000         520.000        27.0
 0.1500        18.032         0.548738         520.000        27.0
 0.4500        17.870         0.491520         520.000        27.0
 0.7500        17.713         0.466102         520.000        27.0
 1.0500        17.551         0.448451         520.000        27.0
 1.3500        17.361         0.402748         520.000        27.0
 1.6500        16.982         0.297437         520.000        27.0
 1.9500        16.459         0.251493         520.000        27.0
 2.2500        15.957         0.250016         520.000        27.0
 2.5500        15.456         0.250004         520.000        27.0
 2.8500        14.956         0.250003         520.000        27.0
 3.0000        14.700         0.250003         520.000        27.0

OIL PRODUCTION IS         4.5586  POUNDS
WATER PRODUCTION IS       0.0209  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS           1.0043
CUMULATIVE MATERIAL BALANCE FOR WATER IS         0.8859
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS        1.0524
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS      0.4480


DISTANCE       PRESSURE        WATER         TEMPERATURE      TIME
  FEET           PSIA        SATURATION           R           HRS.
--------      ---------      ----------      -----------      ----

 0.0000        18.359         1.000000         660.000        27.0
 0.1500        18.353         0.544759         618.449        27.0
 0.4500        18.286         0.473977         580.399        27.0
 0.7500        18.269         0.457646         553.450        27.0
 1.0500        18.113         0.435225         536.740        27.0
 1.3500        17.889         0.371400         527.684        27.0
 1.6500        17.385         0.270977         523.255        27.0
 1.9500        16.793         0.250331         521.252        27.0
 2.2500        16.201         0.250006         520.435        27.0
 2.5500        15.603         0.250004         520.137        27.0
 2.8500        15.004         0.250003         520.027        27.0
 3.0000        14.700         0.250003         520.027        27.0

OIL PRODUCTION IS         4.0482  POUNDS
WATER PRODUCTION IS       0.0186  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS           1.0309
CUMULATIVE MATERIAL BALANCE FOR WATER IS         0.8073
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS        0.9775
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS      0.4944
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DISTANCE       PRESSURE          WATER          TEMPERATURE       TIME
  FEET           PSIA          SATURATION            R            HRS.
--------       ---------       ----------       -----------       ----

 0.0000         17.197          1.000000          520.000         36.0
 0.1500         17.126          0.566632          520.000         36.0
 0.4500         16.991          0.503133          520.000         36.0
 0.7500         16.864          0.474316          520.000         36.0
 1.0500         16.726          0.457186          520.000         36.0
 1.3500         16.585          0.436486          520.000         36.0
 1.6500         16.407          0.368119          520.000         36.0
 1.9500         16.046          0.268540          520.000         36.0
 2.2500         15.655          0.250277          520.000         36.0
 2.5500         15.273          0.250008          520.000         36.0
 2.8500         14.892          0.250005          520.000         36.0
 3.0000         14.700          0.250005          520.000         36.0

OIL PRODUCTION IS         5.1734  POUNDS
WATER PRODUCTION IS       0.0237  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS          1.0138
CUMULATIVE MATERIAL BALANCE FOR WATER IS        0.7607
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS       1.1116
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS     0.3544
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DISTANCE       PRESSURE          WATER          TEMPERATURE       TIME
  FEET           PSIA          SATURATION            R            HRS.
--------       ---------       ----------       -----------       ----

 0.0000         16.487          1.000000          660.000         36.0
 0.1500         16.493          0.564733          624.206         36.0
 0.4500         16.439          0.473330          589.351         36.0
 0.7500         16.455          0.464989          562.738         36.0
 1.0500         16.366          0.447393          544.181         36.0
 1.3500         16.312          0.422002          532.719         36.0
 1.6500         16.156          0.337605          526.287         36.0
 1.9500         15.844          0.257394          522.910         36.0
 2.2500         15.522          0.250084          521.228         36.0
 2.5500         15.197          0.250007          520.405         36.0
 2.8500         14.868          0.250005          519.828         36.0
 3.0000         14.700          0.250005          519.828         36.0

OIL PRODUCTION IS         4.7188  POUNDS
WATER PRODUCTION IS       0.0216  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS          1.0216
CUMULATIVE MATERIAL BALANCE FOR WATER IS        0.6990
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS       0.9564
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS     0.2672
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DISTANCE        PRESSURE         WATER          TEMPERATURE        TIME
  FEET            PSIA         SATURATION            R             HRS.
--------       ---------       ----------       -----------        ----

 0.0000         16.624          1.000000          520.000          45.0
 0.1500         16.567          0.580891          520.000          45.0
 0.4500         16.449          0.512701          520.000          45.0
 0.7500         16.344          0.481197          520.000          45.0
 1.0500         16.234          0.463497          520.000          45.0
 1.3500         16.110          0.447969          520.000          45.0
 1.6500         16.007          0.412629          520.000          45.0
 1.9500         15.800          0.313732          520.000          45.0
 2.2500         15.477          0.253042          520.000          45.0
 2.5500         15.166          0.250035          520.000          45.0
 2.8500         14.856          0.250007          520.000          45.0
 3.0000         14.700          0.250007          520.000          45.0

OIL PRODUCTION IS          5.6776  POUNDS
WATER PRODUCTION IS        0.0260  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS          1.0222
CUMULATIVE MATERIAL BALANCE FOR WATER IS        0.6731
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS       1.1127
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS     0.2889


DISTANCE        PRESSURE         WATER          TEMPERATURE        TIME
  FEET            PSIA         SATURATION            R             HRS.
--------       ---------       ----------       -----------        ----

 0.0000         15.419          1.000000          660.000          45.0
 0.1500         15.427          0.581523          628.169          45.0
 0.4500         15.381          0.468381          595.953          45.0
 0.7500         15.404          0.468260          570.083          45.0
 1.0500         15.349          0.448852          550.733          45.0
 1.3500         15.357          0.432258          537.700          45.0
 1.6500         15.331          0.375970          529.666          45.0
 1.9500         15.214          0.274082          524.987          45.0
 2.2500         15.071          0.250424          522.284          45.0
 2.5500         14.927          0.250012          520.531          45.0
 2.8500         14.778          0.250007          518.624          45.0
 3.0000         14.700          0.250007          518.624          45.0

OIL PRODUCTION IS          5.0600  POUNDS
WATER PRODUCTION IS        0.0232  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS          1.0171
CUMULATIVE MATERIAL BALANCE FOR WATER IS        0.5967
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS       0.9297
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS     0.1174
```

COMPUTER OUTPUT III

√ = NABLA = 1.500

Δx = DX = 0.300 feet

Δt = DT = 0.015 Hrs.

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DISTANCE       PRESSURE        WATER         TEMPERATURE      TIME
  FEET           PSIA        SATURATION           R            HRS.
--------      ---------      ----------      -----------      ----

 0.0000        25.448         1.000000         520.000         9.00
 0.1500        25.066         0.482922         520.000         9.00
 0.4500        24.686         0.407252         520.000         9.00
 0.7500        23.791         0.294014         520.000         9.00
 1.0500        22.537         0.251132         520.000         9.00
 1.3500        21.330         0.250006         520.000         9.00
 1.6500        20.124         0.249998         520.000         9.00
 1.9500        18.919         0.249999         520.000         9.00
 2.2500        17.713         0.250000         520.000         9.00
 2.5500        16.508         0.250001         520.000         9.00
 2.8500        15.303         0.250001         520.000         9.00
 3.0000        14.700         0.250001         520.000         9.00

OIL PRODUCTION IS         1.6944  POUNDS
WATER PRODUCTION IS       0.0078  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS          1.0146
CUMULATIVE MATERIAL BALANCE FOR WATER IS        1.0010
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS       0.9995
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS     1.0051
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DISTANCE       PRESSURE        WATER         TEMPERATURE      TIME
  FEET           PSIA        SATURATION           R            HRS.
--------      ---------      ----------      -----------      ----

 0.0000        24.231         1.000000         660.000         9.00
 0.1500        24.054         0.482056         589.177         9.00
 0.4500        23.875         0.402780         545.909         9.00
 0.7500        23.244         0.291069         527.821         9.00
 1.0500        22.163         0.250959         521.930         9.00
 1.3500        21.037         0.250001         520.391         9.00
 1.6500        19.890         0.249998         520.061         9.00
 1.9500        18.737         0.249999         520.001         9.00
 2.2500        17.583         0.250000         519.992         9.00
 2.5500        16.430         0.250001         519.994         9.00
 2.8500        15.276         0.250002         519.998         9.00
 3.0000        14.700         0.250002         519.998         9.00

OIL PRODUCTION IS         1.5879  POUNDS
WATER PRODUCTION IS       0.0073  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS          1.0618
CUMULATIVE MATERIAL BALANCE FOR WATER IS        0.9813
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS       1.0206
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS     0.9813
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 DISTANCE      PRESSURE        WATER          TEMPERATURE     TIME
   FEET          PSIA        SATURATION            R          HRS.
 --------      ---------     ----------      -----------      ----

  0.0000        21.992        1.000000          520.000       18.00
  0.1500        21.815        0.525016          520.000       18.00
  0.4500        21.608        0.474276          520.000       18.00
  0.7500        21.299        0.437938          520.000       18.00
  1.0500        20.795        0.355678          520.000       18.00
  1.3500        19.881        0.262356          520.000       18.00
  1.6500        18.928        0.250148          520.000       18.00
  1.9500        17.989        0.250003          520.000       18.00
  2.2500        17.049        0.250002          520.000       18.00
  2.5500        16.110        0.250003          520.000       18.00
  2.8500        15.170        0.250004          520.000       18.00
  3.0000        14.700        0.250004          520.000       18.00

 OIL PRODUCTION IS          3.1353  POUNDS
 WATER PRODUCTION IS        0.0144  POUNDS
 CUMULATIVE MATERIAL BALANCE FOR OIL IS           1.0163
 CUMULATIVE MATERIAL BALANCE FOR WATER IS         0.9256
 INSTANTANEOUS MATERIAL BALANCE FOR OIL IS        1.0222
 INSTANTANEOUS MATERIAL BALANCE FOR WATER IS      0.8015
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 DISTANCE      PRESSURE        WATER          TEMPERATURE     TIME
   FEET          PSIA        SATURATION            R          HRS.
 --------      ---------     ----------      -----------      ----

  0.0000        21.787        1.000000          660.000       18.00
  0.1500        21.727        0.530240          608.715       18.00
  0.4500        21.646        0.473011          567.159       18.00
  0.7500        21.492        0.440003          541.700       18.00
  1.0500        21.113        0.370825          528.795       18.00
  1.3500        20.282        0.270074          523.175       18.00
  1.6500        19.284        0.250295          521.003       18.00
  1.9500        18.276        0.250001          520.271       18.00
  2.2500        17.257        0.250001          520.058       18.00
  2.5500        16.235        0.250003          520.008       18.00
  2.8500        15.213        0.250004          520.010       18.00
  3.0000        14.700        0.250004          520.010       18.00

 OIL PRODUCTION IS          3.1891  POUNDS
 WATER PRODUCTION IS        0.0146  POUNDS
 CUMULATIVE MATERIAL BALANCE FOR OIL IS           1.0351
 CUMULATIVE MATERIAL BALANCE FOR WATER IS         0.9591
 INSTANTANEOUS MATERIAL BALANCE FOR OIL IS        1.0032
 INSTANTANEOUS MATERIAL BALANCE FOR WATER IS      0.8581
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DISTANCE        PRESSURE          WATER          TEMPERATURE        TIME
  FEET            PSIA          SATURATION           R              HRS.
--------       ---------        ----------       -----------        ----

 0.0000          20.792          1.000000          520.000          27.00
 0.1500          20.664          0.548801          520.000          27.00
 0.4500          20.504          0.494813          520.000          27.00
 0.7500          20.292          0.469773          520.000          27.00
 1.0500          20.020          0.450660          520.000          27.00
 1.3500          19.607          0.397868          520.000          27.00
 1.6500          18.869          0.291021          520.000          27.00
 1.9500          17.915          0.251093          520.000          27.00
 2.2500          16.995          0.250014          520.000          27.00
 2.5500          16.077          0.250005          520.000          27.00
 2.8500          15.159          0.250006          520.000          27.00
 3.0000          14.700          0.250006          520.000          27.00

OIL PRODUCTION IS          4.4801  POUNDS
WATER PRODUCTION IS        0.0205  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS            1.0195
CUMULATIVE MATERIAL BALANCE FOR WATER IS          0.8840
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS         1.0258
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS       0.7855
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DISTANCE        PRESSURE          WATER          TEMPERATURE        TIME
  FEET            PSIA          SATURATION           R              HRS.
--------       ---------        ----------       -----------        ----

 0.0000          19.169          1.000000          660.000          27.00
 0.1500          19.144          0.558247          618.368          27.00
 0.4500          19.088          0.491312          580.697          27.00
 0.7500          19.024          0.463697          553.557          27.00
 1.0500          18.891          0.446138          536.784          27.00
 1.3500          18.643          0.401646          527.686          27.00
 1.6500          18.107          0.297491          523.248          27.00
 1.9500          17.351          0.251512          521.248          27.00
 2.2500          16.602          0.250014          520.431          27.00
 2.5500          15.844          0.250004          520.136          27.00
 2.8500          15.083          0.250006          520.033          27.00
 3.0000          14.700          0.250006          520.033          27.00

OIL PRODUCTION IS          4.4768  POUNDS
WATER PRODUCTION IS        0.0205  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS            1.0257
CUMULATIVE MATERIAL BALANCE FOR WATER IS          0.8886
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS         1.0070
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS       0.6425
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DISTANCE          PRESSURE           WATER           TEMPERATURE         TIME
  FEET              PSIA           SATURATION             R              HRS.
--------         ---------        ----------        -----------          ----

 0.0000           19.594           1.000000          520.000            36.00
 0.1500           19.490           0.566113          520.000            36.00
 0.4500           19.358           0.509392          520.000            36.00
 0.7500           19.182           0.481667          520.000            36.00
 1.0500           18.966           0.467165          520.000            36.00
 1.3500           18.715           0.456059          520.000            36.00
 1.6500           18.382           0.423150          520.000            36.00
 1.9500           17.802           0.328592          520.000            36.00
 2.2500           16.899           0.255221          520.000            36.00
 2.5500           16.016           0.250059          520.000            36.00
 2.8500           15.139           0.250008          520.000            36.00
 3.0000           14.700           0.250008          520.000            36.00

OIL PRODUCTION IS         5.7692  POUNDS
WATER PRODUCTION IS       0.0265  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS             1.0206
CUMULATIVE MATERIAL BALANCE FOR WATER IS           0.8544
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS          1.0236
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS        0.7488


DISTANCE          PRESSURE           WATER           TEMPERATURE         TIME
  FEET              PSIA           SATURATION             R              HRS.
--------         ---------        ----------        -----------          ----

 0.0000           17.842           1.000000          660.000            36.00
 0.1500           17.832           0.579027          624.182            36.00
 0.4500           17.788           0.500609          589.796            36.00
 0.7500           17.763           0.471177          562.888            36.00
 1.0500           17.706           0.458223          544.362            36.00
 1.3500           17.549           0.446870          532.800            36.00
 1.6500           17.309           0.404072          526.278            36.00
 1.9500           16.869           0.300452          522.895            36.00
 2.2500           16.249           0.251738          521.227            36.00
 2.5500           15.636           0.250019          520.412            36.00
 2.8500           15.014           0.250007          519.850            36.00
 3.0000           14.700           0.250007          519.850            36.00

OIL PRODUCTION IS         5.4708  POUNDS
WATER PRODUCTION IS       0.0251  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS             1.0219
CUMULATIVE MATERIAL BALANCE FOR WATER IS           0.8111
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS          1.0012
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS        0.5215
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DISTANCE      PRESSURE         WATER        TEMPERATURE        TIME
  FEET          PSIA        SATURATION           R             HRS.
--------     ---------      ----------      -----------        ----

 0.0000        18.584        1.000000         520.000          45.00
 0.1500        18.494        0.579729         520.000          45.00
 0.4500        18.379        0.521356         520.000          45.00
 0.7500        18.228        0.491824         520.000          45.00
 1.0500        18.037        0.474745         520.000          45.00
 1.3500        17.818        0.465366         520.000          45.00
 1.6500        17.575        0.458318         520.000          45.00
 1.9500        17.284        0.437623         520.000          45.00
 2.2500        16.810        0.363192         520.000          45.00
 2.5500        15.999        0.265939         520.000          45.00
 2.8500        15.128        0.250295         520.000          45.00
 3.0000        14.700        0.250292         520.000          45.00

OIL PRODUCTION IS          7.0125  POUNDS
WATER PRODUCTION IS        0.0322  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS            1.0210
CUMULATIVE MATERIAL BALANCE FOR WATER IS          0.8310
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS         1.0225
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS       0.7286




DISTANCE      PRESSURE         WATER        TEMPERATURE        TIME
  FEET          PSIA        SATURATION           R             HRS.
--------     ---------      ----------      -----------        ----

 0.0000        16.649        1.000000         660.000          45.00
 0.1500        16.647        0.595894         628.126          45.00
 0.4500        16.610        0.504418         596.326          45.00
 0.7500        16.604        0.474252         570.198          45.00
 1.0500        16.590        0.462215         551.004          45.00
 1.3500        16.495        0.455117         537.903          45.00
 1.6500        16.375        0.443791         529.708          45.00
 1.9500        16.177        0.387565         524.954          45.00
 2.2500        15.824        0.282157         522.280          45.00
 2.5500        15.379        0.250690         520.557          45.00
 2.8500        14.930        0.250013         518.678          45.00
 3.0000        14.700        0.250013         518.678          45.00

OIL PRODUCTION IS          6.2423  POUNDS
WATER PRODUCTION IS        0.0286  POUNDS
CUMULATIVE MATERIAL BALANCE FOR OIL IS            1.0187
CUMULATIVE MATERIAL BALANCE FOR WATER IS          0.7377
INSTANTANEOUS MATERIAL BALANCE FOR OIL IS         0.9939
INSTANTANEOUS MATERIAL BALANCE FOR WATER IS       0.3688
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